Finding the Neutron Flux in a Heterogeneous Core
Detailed Core Geometry

• Typical Heterogeneity in a LWR Core and Assembly Configurations
Heterogeneity of a CANDU Core
Sizing-up the Problem

• Core is very heterogeneous.
• Ideally, we should divide the core into very many small homogeneous regions and solve the multigroup transport equation for the entire core to find the flux in each of these regions

\[ \phi_{rg} \]

• This would involve a very large number of unknowns.
Estimate of the Number of Unknowns

- Consider a typical CANDU core
- At a minimum, we would like to know the flux in each fuel pin, in the coolant in each bundle, and in the moderator corresponding to each bundle.
- Because the flux and hence reaction rates also depend strongly on energy, we also need a fine energy discretization, say 100 groups.
Number of unknowns

\[ N_u = 380 \text{channels} \times 12 \frac{\text{bundles}}{\text{channel}} \times \]
\[ \times 39 \text{regions} (37 \text{pins} + \text{coolant} + \text{moderator}) \times \]
\[ \times \frac{100 \text{unknowns} (\text{group} - \text{fluxes})}{\text{bundle}} \times \]
\[ \times \frac{100 \text{unknowns} (\text{group} - \text{fluxes})}{\text{region}} = 17,784,000 \text{unknowns} \]

Too many even for today's "smart" computers.
**Solution:** Solve an approximate problem. Homogenize (i.e. average over) each *node*, condense (collapse) the energy groups and then find the flux for this simplified model.

A *node* corresponds to one lattice cell by one bundle length in CANDU or one fuel assembly by some arbitrary length in LWR.
Cell/Node-Homogenized Core Geometry
(Assembly-size Cell/Node)
Finding the Node-Homogenized Macroscopic Cross Sections - Lattice Calculations

1. Discretize the node geometry into small spatial regions.
2. Assign macroscopic cross sections to each fine region based on its composition and on the microscopic cross sections of its constituents.
3. Solve the multigroup (approx. 80 groups) transport equation in the node using a fine spatial discretization and fine energy-group structure, assuming zero neutron current on the boundary of the node (approximation).
4. Average the cross sections over the regions in the node and condense (collapse) them onto few (2-4) energy groups.
Example of CANDU Node Discretization
(Cell assumed infinite in the Z direction - 2D calculation)
Note that, in fact, there is more than one region per fuel pin.
Calculation of Macroscopic Cross Sections for Each Region

\[ \sum_{rg}^{x} = \sum_{i=1}^{n_r} N_{ri} \sigma_{ig}^{x} \]

\( r \) = region index
\( g \) = energy-group index
\( x \) = reaction index (e.g. fission, capture, scattering, etc.)
\( n_r \) = number of different isotopes in region \( r \)
\( N_{ri} \) = atom (number) density of isotope \( i \) in region \( r \).
\( \sigma_{ig}^{x} \) = microscopic cross section of reaction \( x \) for isotope \( g \) in group \( g \)
Lattice (Assembly/Bundle) Calculation

- Performed using a transport code, fine geometric detail and many energy groups
- Results
  - The flux in each region, for each fine energy group. (The fine-energy group flux is also known as the many-group flux).

\[ \varphi_{rg} \]
Flux and Cross Section Condensation (Collapsation) in Each Fine Region

Definition of condensation (collapsation)

\[ \Phi_G = \sum_{g \in G} \varphi_g \]

\[ \bar{\Sigma}_G = \frac{\sum_{g \in G} \varphi_g}{\Phi_G} \]

- The overbar denotes average over several fine energy groups.
- Note that the condensed flux is the \textit{sum} (not the average) of several fine-group fluxes. That’s why we don’t use an overbar for it.
For each region $r$ we then have:

$$
\Phi_{rG} = \sum_{g \in G} \varphi_{rg}
$$

$$
\sum_{rG}^x = \frac{\sum_{g \in G} \sum_{rG}^x \varphi_{rg}}{\Phi_{rG}}
$$

The coarse-group flux is called the \textit{few-group flux}.
Flux and Cross Section Homogenization

\[ V_R = \sum_{r \in R} V_r \]

\[ \hat{\Phi}_{rG} = \frac{\sum_{r \in R} V_r \Phi_{rG}}{V_R} \]

\[ \hat{\sum}_{rG}^{x} = \frac{\sum_{r \in R} \sum_{rG}^{x} \Phi_{rG} V_r}{\hat{\Phi}_{rG} V_R} \]

- The hat denotes average over regions.
Homogenized and condensed (collapsed) flux and cross sections

\[ \hat{\Phi}_{RG} = \frac{\sum_{r \in R} V_r \sum_{g \in G} \phi_{rg}}{V_R} \]

\[ \hat{\sum}_x^{RG} = \frac{\sum_{r \in R} \sum_{g \in G} \sum_x \phi_{rg} V_r}{\Phi_{RG} V_R} \]
Core Homogenization

For situations when only rough results are necessary, an additional simple homogenization can be performed for the entire core. A simple flux-weighted volume average is performed over all nodes (indexed by R).

\[ \tilde{\sum}_x^{\sum_{core \ G}} \approx \sum_{R \in \text{core}} \sum_{RG}^x \Phi_{RG} V_R \]

\[ \sum_R \Phi_{RG} V_R \]

The core is now approximated to be homogeneous and all the methods we described for homogeneous cores apply.
Core Calculation

Employs a simplified (node-homogenized) core model.

Is performed using the diffusion approximation:

\[ \vec{J}_G = -D \nabla \Phi_G \]

Usually employs a finite-difference or nodal solution method.
Node-Fuel Properties

 Depend on:
  ➢ Fuel composition
  ➢ Local parameters
    ▪ Fuel temperature
    ▪ Coolant density
    ▪ Coolant temperature
    ▪ Moderator density
    ▪ Moderator temperature
Fuel Composition

- Depends on how long the fuel has been in the reactor and in how high a flux.
- There are two parameters than can be used to define the level of irradiation of the fuel:
  - Specific Burnup (or, simply, burnup)
  - Irradiation
Conversion and Burnup of Fuel

Capture is the reaction: \( ^A X + n \rightarrow ^{A+1} X + \gamma \)

Capture-to-fission ratio

\[
\alpha = \frac{\sigma_\gamma}{\sigma_f}
\]

Number of neutrons released per absorbed neutron.

\[
\eta = \nu \frac{\sigma_f}{\sigma_a}
\]

For mixtures of fissile and non-fissile elements:

\[
\eta = \frac{1}{\Sigma_a} \sum_i \nu_i \Sigma_{fi}
\]
Fissile Nuclei

Nuclei that have a fairly large fission cross section at zero energy.

Fissile nuclei can fission by reacting with a neutron of almost zero kinetic energy (slow/thermal neutron)

Example of fissile nuclei: $^{235}\text{U}$, $^{239}\text{Pu}$, $^{233}\text{U}$

$^{235}\text{U}$ is the only naturally-occurring fissile nucleus
Fissionable Nuclei

Nuclei that can undergo fission (either by a slow or a fast neutron) are called fissionable. Those that can undergo fission by a slow neutron are called fissile (as discussed).

In some fissionable nuclei the energy-dependent fission cross section displays a threshold. It is near-zero up to the threshold and fairly large above the threshold.

_Such nuclei which can fission only by reacting with a neutron of high kinetic energy (fast neutron) are fissionable but not fissile._

Example of fissionable nuclei that are not fissile: $^{232}\text{Th}$, $^{238}\text{U}$
Fertile Nuclei

Nuclei that can capture a neutron and mutate into a fissile nucleus.

Example of Fertile Nuclei: $^{232}\text{Th}$, $^{238}\text{U}$
(Happen to be fissionable as well)

\[ ^{232}\text{Th}(n,\gamma)^{233}\text{Th} \rightarrow ^{233}\text{Pa} \rightarrow ^{233}\text{U} \]

\[ ^{238}\text{U}(n,\gamma)^{239}\text{U} \rightarrow ^{239}\text{Np} \rightarrow ^{239}\text{Pu} \]
Fuel Burnup

Specific Burnup (sometimes called just Burnup)

\[
B = \frac{Fission\ Energy\ produced}{Total\ Initial\ Mass\ of\ fuel}
\]

NOTE: “Fuel” includes fissile and fertile nuclides only (heavy elements)

Approximate Maximum Specific Burnup

\[
\frac{200\ MeV}{235\ amu} = 950,000\ \frac{MWd}{t}
\]
Fuel Irradiation

 Equals the time integral of the fuel flux (fluence)

$$\omega = \int_{0}^{t} \Phi(\tau) d\tau$$
Modelling of Reactivity Devices

In principle, transport calculations should be performed for every lattice cell that includes a reactivity device.

This is a complicated 3-D transport calculation.

To save time, this calculation is performed only once for each type of reactivity device and for an irradiation roughly equal to half of the discharge irradiation.
Modelling of Reactivity Devices

Obviously, the cell-averaged cross sections in the presence of reactivity devices are different from those in their absence.

Cross section increments are then calculated as:

\[ \delta \Sigma = \Sigma_{cell}^{device} - \Sigma_{cell}^{0} \]

Once increments are calculated, they are added to all nodes that are affected by reactivity devices, regardless of the irradiation of the fuel in that node.
Core Analysis Steps

1. Lattice Calculations
   1.1. Find the approximate flux in each node ("cell" in CANDU parlance) assuming reflective BC
   1.2. Calculate node-homogenized multigroup cross-sections

2. Core calculation
   2.1. Find the flux in the entire core using an approximate, core-homogenized model.
Specific Features for Continuous Refuelling

- Node-homogenized cross sections depend on the bundle irradiation (same thing as fluence).

\[ \omega = \int_{0}^{t} \Phi(\tau) d\tau \]

- (Since no confusion occurs, no overbar or hat is used for the node-homogenized cross sections).

\[ \Sigma_{G}^{x} = \Sigma_{G}^{x}(\omega) \]
• Initially, all fuel in the core is fresh. This is called the fresh core.
• As the fuel burns, refueling begins.
• Since different bundles in the core have different irradiation values, their cross sections will be different.
• After a long period of refueling the core reaches a somewhat constant distribution of irradiation. This situation is known as the equilibrium core.
Coarse Approximation for Equilibrium Core Properties

We will assume, for the purpose of finding the distribution of irradiation, the following:

- The core is homogeneous.
- The homogeneous properties of the core are found by simple flux-weighted volume averaging of the local (bundle) properties.
- Refuelling of each channel is continuous, the fuel being pushed in at a constant speed \( u \). (This is not realistic, as we cannot refuel fractions of a bundle, but it is a useful approximation).
- For each channel, the speed is chosen such that the discharge irradiation is the same for all channels and equal to \( \omega_{\text{disch}} \).
- The flux varies axially in each channel and also varies from channel to channel, but consistent with the discharge irradiation being the same in each channel.
Coarse Approximation for Equilibrium Core Properties

Averaging the cross sections for one channel, we obtain:

$$
\overline{\Sigma}_{\text{channel}} = \frac{\int_0^L \Sigma(z) \Phi(z) \, dz}{\int_0^L \Phi(z) \, dz} = \frac{\int_0^L \Sigma(\omega(z)) \Phi(z) \, dz}{\int_0^L \Phi(z) \, dz}
$$

The irradiation is:

$$
\omega(z) = \int_0^{t(z)} \Phi(\tau) \, d\tau = \int_0^z \Phi(\zeta) \frac{d\tau}{d\zeta} \, d\zeta = \frac{1}{u} \int_0^z \Phi(\zeta) \, d\zeta
$$
Coarse Approximation for Equilibrium Core Properties

The discharge irradiation is:

\[\omega_{\text{disch}} = \omega(L) = \frac{1}{u} \int_{0}^{L} \Phi(\zeta) d\zeta\]

From the definition of irradiation, it follows that:

\[\frac{d\omega(z)}{dz} = \frac{1}{u} \Phi(z)\]
Hence, our average cross section is:

\[
\overline{\Sigma}_{\text{channel}} = \frac{\int_0^L \Sigma(\omega(z))\Phi(z)dz}{\int_0^L \Phi(z)dz} = \frac{\int_0^L \Sigma(\omega(z))u \frac{d\omega(z)}{dz}dz}{\int_0^L u \frac{d\omega(z)}{dz}dz} = \frac{\int_0^{\omega_{\text{disch}}} \Sigma(\omega)d\omega}{\int_0^{\omega_{\text{disch}}} d\omega} = \frac{\omega_{\text{disch}}}{\omega_{\text{disch}}}
\]

Since the above is the same for every channel (we assumed \(\omega_{\text{disch}}\) was the same), averaging it over all channels will produce the same value. So:

\[
\overline{\Sigma}_{\text{core}} (\omega_{\text{disch}}) = \frac{\int_0^{\omega_{\text{disch}}} \Sigma(\omega)d\omega}{\omega_{\text{disch}}}
\]
Calculating the Discharge Irradiation

Based on the (approximately homogeneous) core cross sections, we can calculate the core $k_{\text{eff}}$.

For example, in a one-group approximation:

$$k_{\text{eff}} = \frac{\nu \Sigma_f}{DB^2_{\text{geo}} + \sum_a}$$

But all core-average cross sections depend on the discharge burnup.

$$k_{\text{eff}}(\omega_{\text{disch}}) = \frac{\nu \Sigma_f (\omega_{\text{disch}})}{DB^2_{\text{geo}} + \sum_a (\omega_{\text{disch}})}$$
Calculating the Discharge Irradiation

Since the reactor needs to be critical, the discharge irradiation is found by solving:

\[
\kappa_{\text{eff}}(\omega_{\text{disch}}) = \frac{\nu \Sigma_f(\omega_{\text{disch}})}{DB^2_{\text{geo}} + \Sigma_a(\omega_{\text{disch}})} = 1
\]
Improvements on the One-Group Continuous-Refuelling Homogeneous Model

1. Use two or more energy groups
2. Use a better model for refueling
   2.1. Time-average
   2.2. Core follow