UNENE

Physics Refresher Course

Part 1

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Textbook

Introduction to Nuclear Engineering (third edition)
J.R. Lamarsh & A.J. Baratta
Prentice-Hall, 2001
ISBN: 0-201-82498-1

Outline ***(Italics denote corresponding textbook chapter and section.)***

1. Atomic And Nuclear Physics
   - 1.1. Elements of Relativity ***(2.5)***
     - 1.1.1. Relativistic Mass Formula
     - 1.1.2. Relativistic Energy
     - 1.1.3. Relativistic Momentum
   - 1.2. Photoelectric Effect ***(3.8)***
   - 1.3. Compton Effect ***(3.8)***
   - 1.4. Atomic Spectra ***(no textbook section)***
   - 1.5. Bohr's Atomic Model ***(no textbook section)***
   - 1.6. De Broglie Waves ***(2.6)***

1.7. Atomic and Nuclear Structure
  1.7.1. Atomic and Nuclear Constituents ***(2.1, 2.2)***
  1.7.2. Notations of Isotopes ***(2.3)***
  1.7.3. Descriptions of Nuclear Particles (Mass, Charge, Spin) ***(2.1)***
  1.7.4. Properties and structure of Nuclei ***(2.4)***
  1.7.5. Binding Energy ***(2.11)***
  1.7.6. Nuclear Models ***(2.7, 2.12)***
1.8. Nuclear Reactions ***(2.10)***
  1.8.1. Conservation laws
  1.8.2. Q value

2. Radioactivity ***(2.8, 2.9)***
    2.1. The Decay Process
    2.2. Natural Radioactivity
    2.3. Induced Radioactivity (activation)
    2.4. Decay Chains
3. Interaction of Radiation With Matter
    3.1. Atom Density ***(2.14)***
    3.2. Interactions of Heavy Charged Particles ***(3.9)***
    3.3. Interactions of Light Charged Particles ***(3.9)***
    3.4. Interactions of Gamma Radiation ***(3.8)***
    3.5. Interactions of Neutrons ***(3.1)***
        3.5.1. Types of Neutron-Induced Nuclear Reactions
    3.6. Reaction Cross Sections ***(3.2)***
    3.7. Attenuation of a parallel beam ***(3.3)***
    3.8. Reaction Rate Density ***(3.3)***

Main Results of Special Relativity

Special Relativity – Formulas

- Relativistic mass

$$m = \frac{1}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} m_0$$

- Relativistic momentum

$$\vec{p} = m\vec{\mathrm{v}} = \frac{1}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} m_0 \vec{\mathrm{v}}$$

- Relativistic (total) energy

$$E = mc^2$$

- Relativistic kinetic energy

$$KE = mc^2 - m_0 c^2$$

Quantum Properties of Matter and Light

Is light a wave or is it made up of particles?

• Newton
– particles (cites reflection and propagation in straight line)

• Huyghens
– wave (cites interference, refraction, diffraction)

Beginning of 20th Century

• The wave theory of light was prevalent as it seemed to explain all phenomena involving light.
– reflection
– interference (diffraction as well)

• Moreover, Maxwell had shown light to be an electromagnetic wave (as were X rays).

• Huyghens seemed to have won the dispute, but....

A few phenomena could not be explained by the wave theory

- Black body radiation
- Photoelectric effect
- Compton effect
- Atomic spectra

Black Body Radiation

• A “black body” is a body that only absorbs and emits light, but does not reflect it.
• A black body emits light with a continuous spectrum.
• Attempts to explain theoretically the shape of the spectrum of the black body radiation based on classical theory had failed, especially for small wavelengths.
• Max Planck was able to explain the entire spectrum, by assuming that energy could only be absorbed or emitted in discrete units called **quanta**.

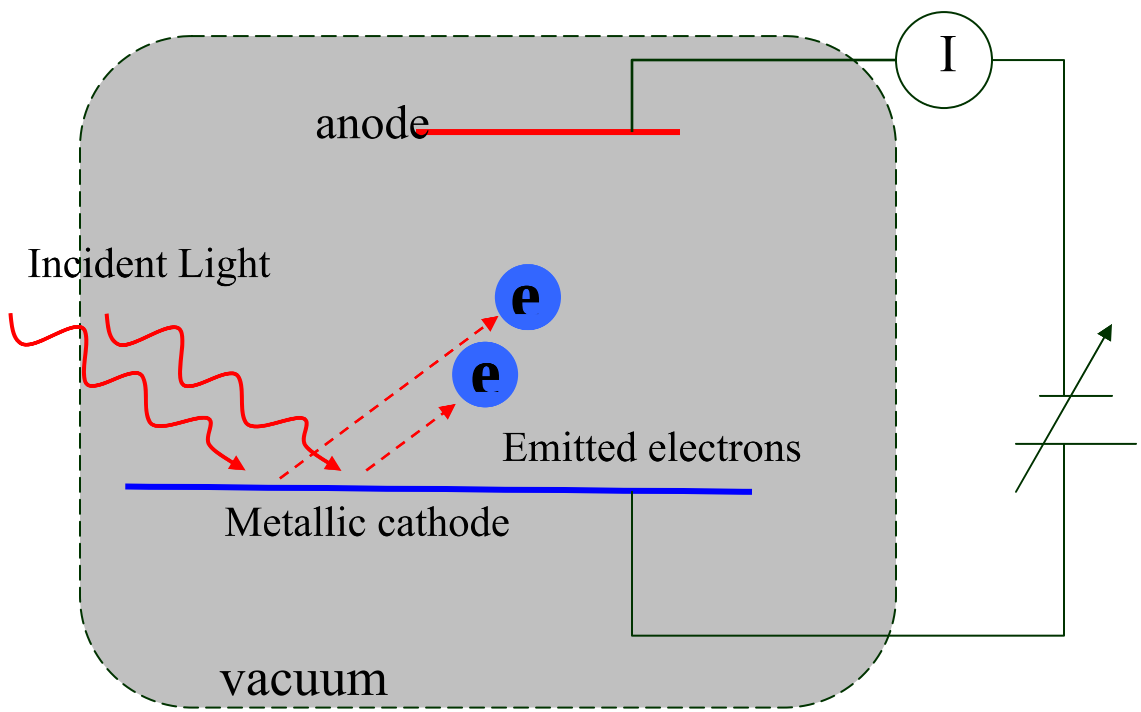
Max Planck – Light Quanta

- Energy of one quantum $E = hf$
- Planck's Constant $h = 6.626 \times 10^{-34} J \cdot s$
- Same dimensions as angular momentum
- Very small number

Photoelectric Effect

Photoelectric Effect

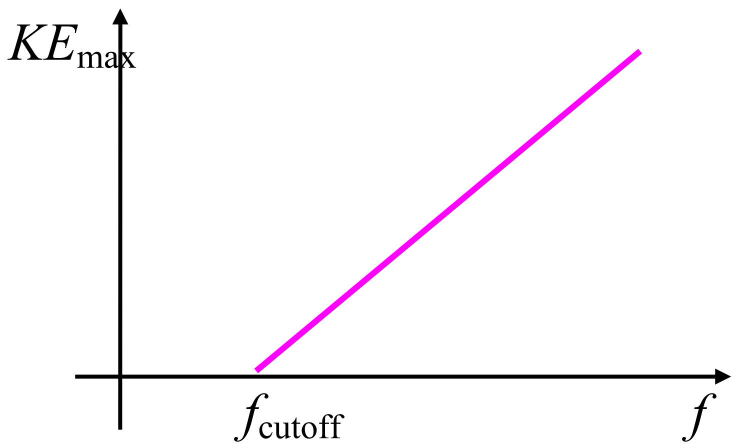
- When light is incident on the (metallic) cathode, electrons are emitted. (called photoelectrons)



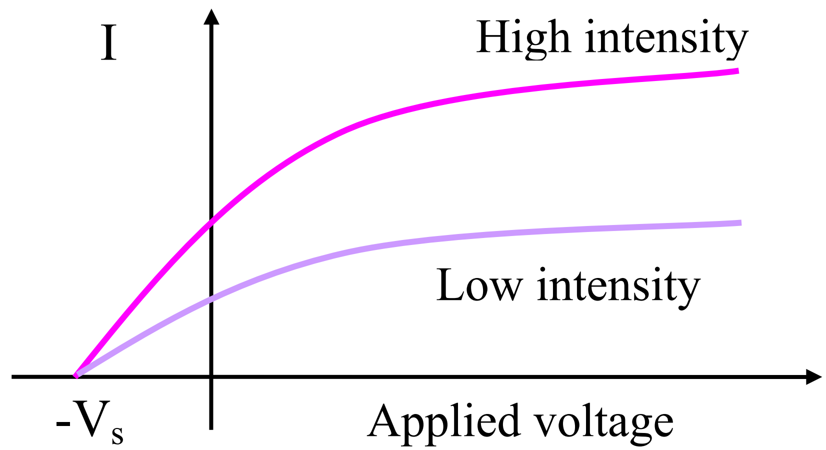
Laws of the Photoelectric Effect

1. No electrons emitted if the frequency of the incident light is lower than a certain value, called the "cutoff frequency".

2. The maximum KE of electrons increases linearly with light frequency.



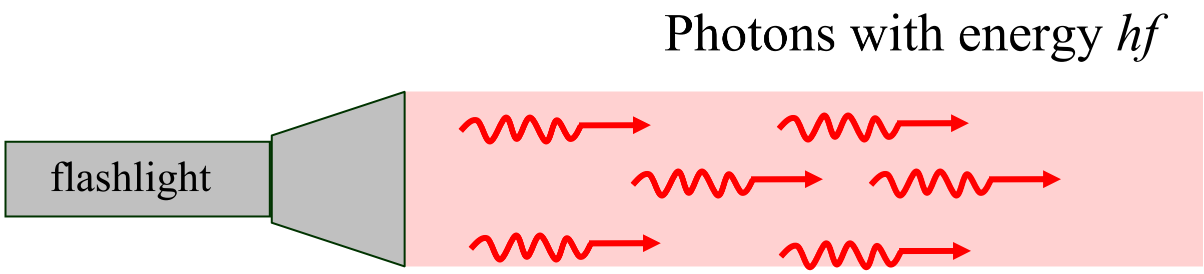
3. Above the cutoff frequency, the maximum number of photoelectrons is proportional to the light intensity.



4. Electrons are emitted almost instantaneously (10^{-9} s after beginning of illumination) although the classical electromagnetic theory would predict some delay.

Einstein's Theory of the PE Effect

1. A light beam consists of quanta (photons), each of energy (Planck's hypothesis) $E = hf$, traveling at the speed of light, c



2. Each photon gives **all** its energy instantaneously to an electron in the metallic cathode.

- If, and only if, the photon's energy is higher than the minimum binding energy in the metal (called the **work function**, Φ), an electron is emitted.
- Consequently, the maximum kinetic energy of an electron is:

$$KE_{\max} = hf - \Phi$$

Values of the work function for selected metals

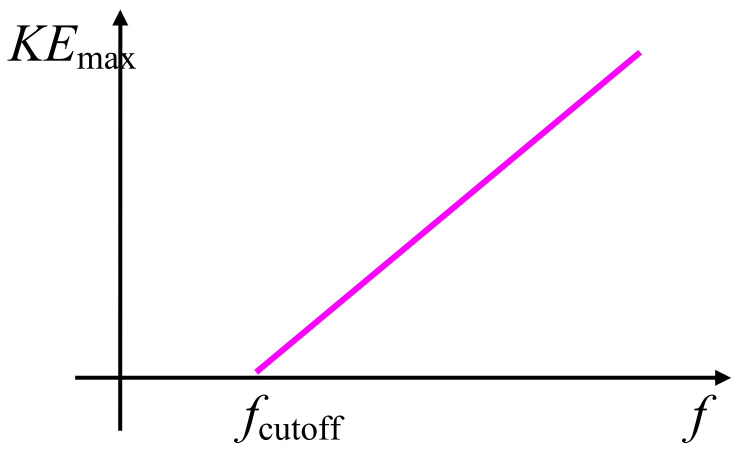
Metal	Φ(eV)
Na	2.28
Al	4.08
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Fe	4.14
Pb	4.50

How Einstein's theory explains the four laws of the PE effect

1. Since $KE_{\max} = hf - \Phi$ has to be positive for the electron to be emitted, it follows that nothing happens below a cutoff frequency.

$$KE_{\max} > 0 \Leftrightarrow f > \frac{\Phi}{h} = f_{cutoff}$$

2. $KE_{\max} = hf - \Phi$ describes exactly the linear relationship between the maximum kinetic energy and light frequency that was found experimentally.



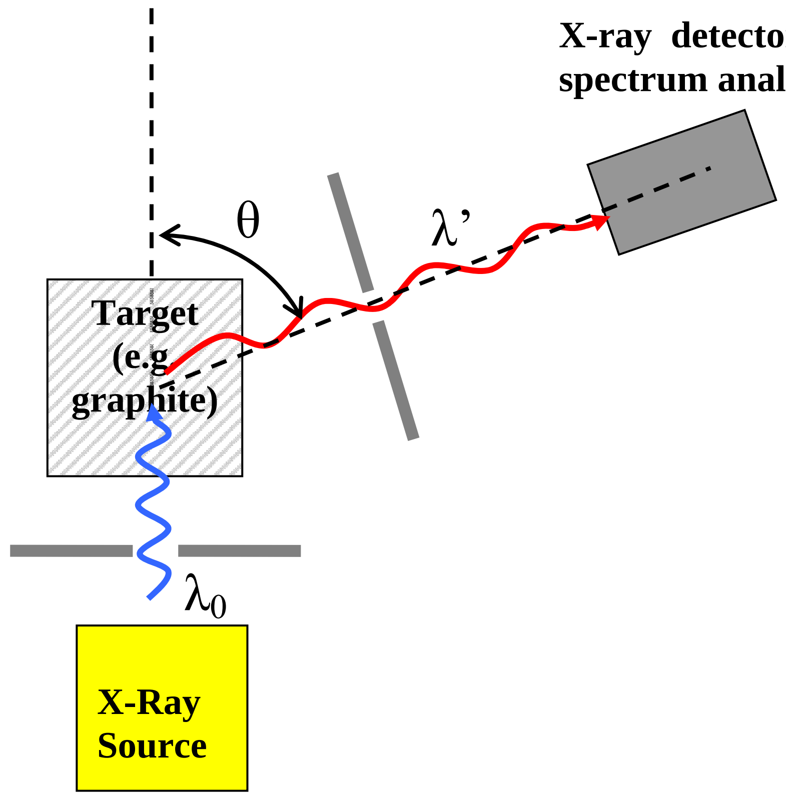
3. Since each photon has the same energy, equal to hf , the intensity of the light is proportional to the number of incident photons per unit time. Since each photon transfers its energy to one electron, it follows that the number of emitted photoelectrons is proportional to the intensity of the incident light.

4. Since each photon interacts with a single electron, the energy transfer happens instantaneously, rather that over a period of time, as would be the case if energy was distributed uniformly in the wave.

Compton Effect

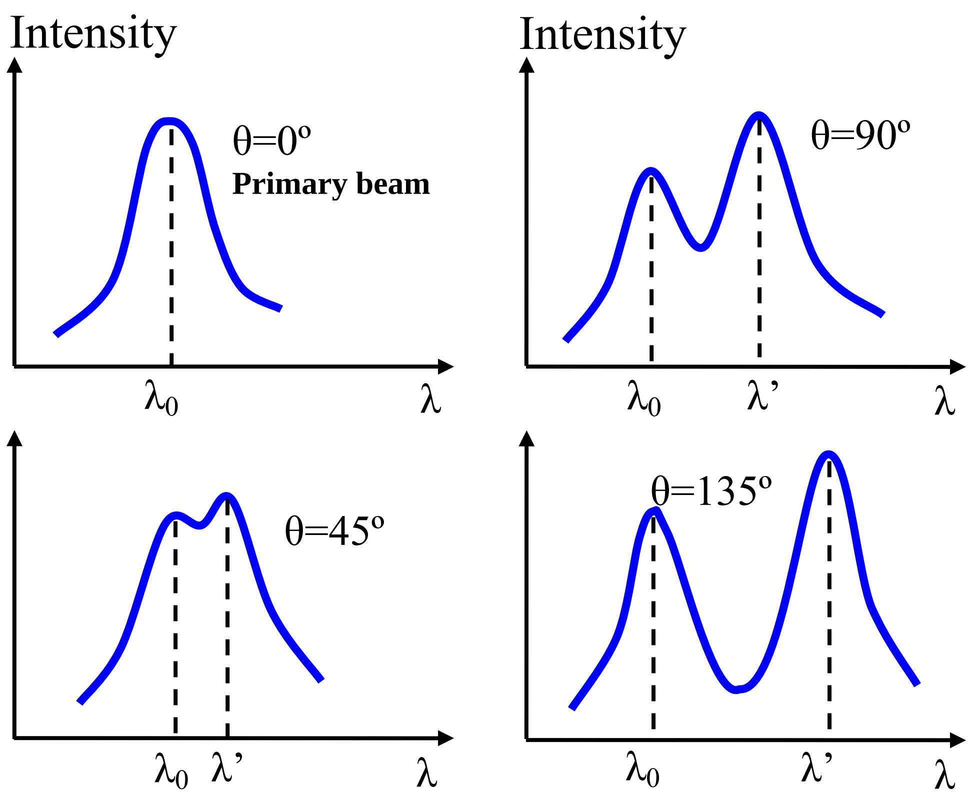
The Compton Effect

- Named after Arthur H. Compton (1892-1962)
- Interaction of electromagnetic radiation with "free" electrons.



Compton's Measurements

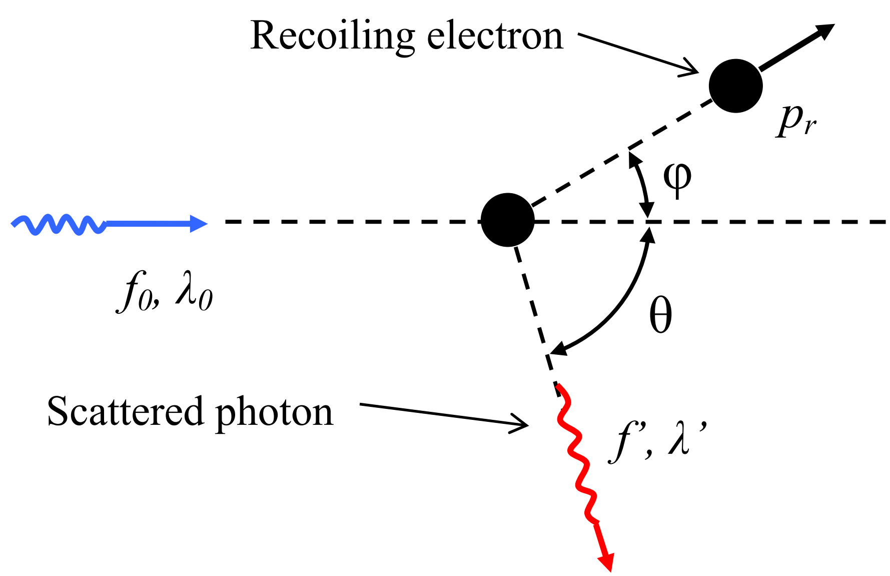
- Frequency of scattered radiation depends only on scattering angle.



Possible Explanations

- Classical Electromagnetic Theory – Inadequate
  - Frequency of scattered radiation depends on beam intensity and time of exposure.

- Compton's Theory
  - Photons undergo elastic collisions with "free" electrons.



• To explain the shift in wavelength, the laws of conservation of **relativistic** energy and momentum need to be applied.

$$E_{0ph} + E_{0e} = E'_{ph} + E'_{e}$$

$$\vec{p}_{0ph} + \vec{p}_{0e} = \vec{p}'_{ph} + \vec{p}'_{e}$$

• Final result:

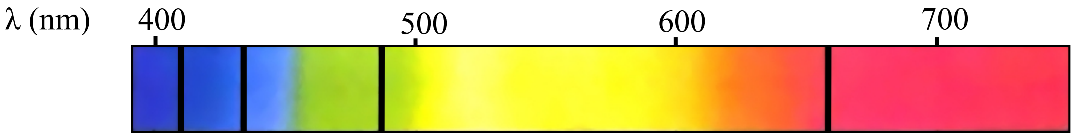
$$\lambda' - \lambda_0 = \frac{h}{m_{0e}c}\left(1 - \cos\theta\right)$$

Atomic Spectra

Atomic Spectra

- Emission Spectrum of H

- Absorption Spectrum of H



- Atomic spectra are discrete (appear as lines)

Hydrogen Spectrum

The wavelengths of emitted/absorbed electromagnetic radiation were found (empirically) to satisfy an interesting relationship:

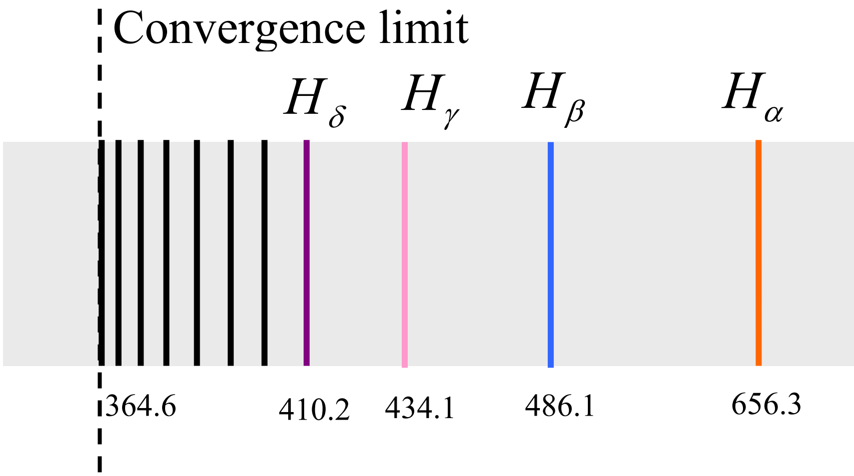
$$\frac{1}{\lambda} = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \quad n > m$$

R_H is called Rydberg's Constant

Hydrogen Spectrum cont.

- All lines obtained for a given m, are said to form a series.
- Balmer series, n=2 – first one discovered (Johann Balmer)

$$\frac{1}{\lambda_n} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad n > 2$$



Atomic Models

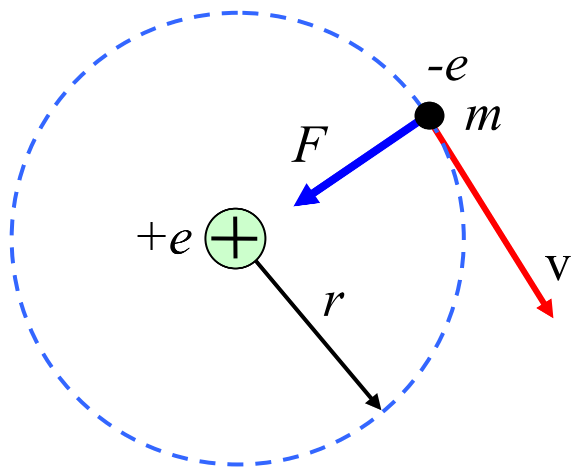
Rutherford's Model of the Hydrogen Atom

- Electrostatic force

$$F_e = \frac{e^2}{4\pi\varepsilon_0 r^2} = k\frac{e^2}{r^2}$$

- Centripetal force

$$F_c = m_e \frac{\mathrm{v}^2}{r}$$



- The two are one and the same

(The electrostatic force acts as centripetal force).

Bohr's Model of the Hydrogen Atom

• The hydrogen atom has only one electron.
• The nucleus consists of only one proton
• Bohr started from Rutherford's model, which assumed the negatively-charged electron to gravitate around the positively-charged proton on a circular orbit.
– The electrostatic attraction force acts as the centripetal force.
• Rutherford's model had limitations
– According to electromagnetic theory, orbiting electrons would radiate light continuously at the frequency they rotated, and in doing so they would lose energy, and eventually fall onto the nucleus.
– This phenomenon was never observed.

Bohr's Model of the Hydrogen Atom (cont.)

- Bohr's additional hypotheses (nonrelativistic)
  - Certain orbits (radii) are stable. No radiative loss of energy occurs for these orbits.
  - The allowed (stable) orbits are those for which the orbital angular momentum has values given by:

$$L = m_e \mathrm{v} r = n\hbar; \quad \hbar \equiv \frac{h}{2\pi}$$

  - Electrons can jump from one orbit to another. Only when such a jump occurs energy is either emitted or absorbed, in the form of a photon.

Bohr's Model of the Hydrogen Atom (cont.)

- Need to find the radius and energy of stable orbits (also called Bohr orbits).
- Start from equating the centripetal force with the electrostatic force

$$m_e \frac{\mathrm{v}^2}{r} = k \frac{e^2}{r^2}$$

- Relationship between radius and speed

$$m_e \frac{\mathrm{v}^2}{r} = k \frac{e^2}{r^2} \Leftrightarrow m_e \mathrm{v}^2 = k \frac{e^2}{r}$$

$$\mathrm{v}^2 = k \frac{e^2}{m_e r}$$

$$\mathrm{v} = \sqrt{\frac{k}{m_e}} \frac{e}{\sqrt{r}}$$

Bohr's Model of the Hydrogen Atom (cont.)

- Expression of angular momentum

$$L = m_e \mathrm{v} r = m_e \left(\sqrt{\frac{k}{m_e}} \frac{e}{\sqrt{r}} \right) r = \sqrt{m_e k} e \sqrt{r}$$

- Use the postulated values of the angular momentum to find the radii of the stable orbits.

$$L_n = n\hbar \Leftrightarrow \sqrt{m_e k} e \sqrt{r_n} = n\hbar$$

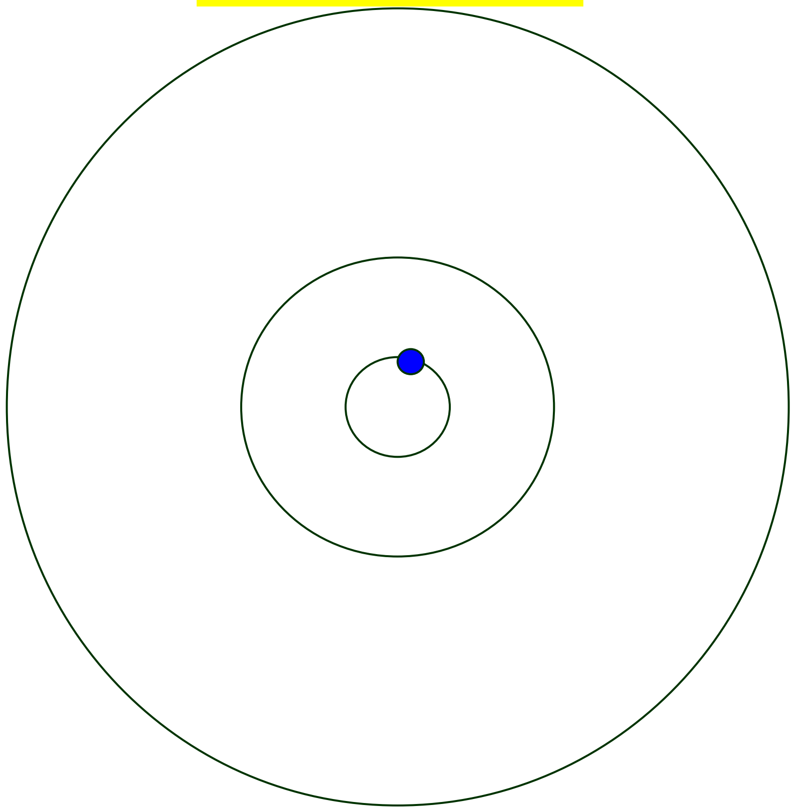
(equation for radii)

Bohr's Model of the Hydrogen Atom (cont.)

- Radii of stable orbits

$$\sqrt{m_e k e}\sqrt{r_n} = n\hbar \Leftrightarrow \sqrt{r_n} = \frac{n\hbar}{\sqrt{m_e k e}}$$

$$r_n = n^2 \frac{\hbar^2}{m_e k e^2}$$



Bohr's Model of the Hydrogen Atom (cont.)

- Energy of electron on a stable orbit

$$E_n = PE_n + KE_n = -\frac{ke^2}{r} + \frac{m_e \mathrm{v}^2}{2}$$

- Substitute previously found expression for v^2

$$\mathrm{v}^2 = k\frac{e^2}{m_e r}$$

- Find simpler expression for energy (not yet final)

$$E_n = -\frac{ke^2}{r_n} + \frac{m_e}{2}k\frac{e^2}{m_e r_n} = -\frac{ke^2}{2r_n}$$

Bohr's Model of the Hydrogen Atom (cont.)

- Final formula for electron's energy on stable orbit

$$E_n = -\frac{ke^2}{2r_n}$$

$$r_n = \frac{n^2\hbar^2}{m_e ke^2}$$

$$E_n = -\frac{ke^2}{2\dfrac{n^2\hbar^2}{m_e ke^2}} = -\frac{m_e k^2 e^4}{2n^2\hbar^2}$$

$$E_n = -\frac{1}{n^2}\frac{m_e k^2 e^4}{2\hbar^2}$$

Bohr's Model of the Hydrogen Atom (cont.)

- Transitions (jumps)

– When an electron jumps from one orbit to another, it has to either absorb or emit energy, in the form of a photon.

  – The energy of the photon equals the difference between the energies of the two orbits. For an electron jumping from orbit n to orbit m, we have:

$$hf = E_n - E_m = \frac{1}{m^2}\frac{m_e k^2 e^4}{2\hbar^2} - \frac{1}{n^2}\frac{m_e k^2 e^4}{2\hbar^2} = \frac{m_e k^2 e^4}{2\hbar^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

– If n>m, the potential energy of the initial state (n) is larger than that of the final state (m) and energy is emitted in the form of a photon. If m>n, the situation is reversed and a photon needs to be absorbed.

Bohr's Model of the Hydrogen Atom (cont.)

- Transitions

– expressing the reciprocal of the wavelength of the photon: $\frac{1}{\lambda} = \frac{f}{c}$

$$hf = \frac{m_e k^2 e^4}{2\hbar^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \Leftrightarrow \frac{f}{c} = \frac{1}{\lambda} = \frac{m_e k^2 e^4}{2\hbar^2 hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$\times \frac{1}{hc}$ $\times \frac{1}{hc}$

$$\frac{1}{\lambda} = \frac{m_e k^2 e^4}{2\hbar^2 hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

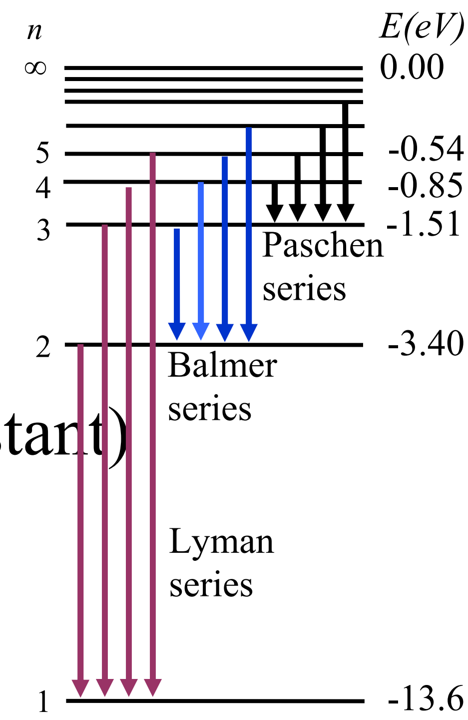
- Previously discovered by Balmer for m=2

Bohr's Model of the Hydrogen Atom (cont.)

- Transitions

$$\frac{1}{\lambda} = \frac{m_e k^2 e^4}{2\hbar^2 hc}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) = R_H\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

$$\frac{m_e k^2 e^4}{2\hbar^2 hc} = R_H \text{ (Rydberg's constant)}$$



Wave-Particle Duality

Light: Wave or Particle?

- Wave
  - Reflection
  - Refraction
  - Interference
  - Diffraction

- Particle
  - Black body radiation
  - Photoelectric effect
  - Compton effect
  - Discrete (line) atomic spectra

- Conclusion
  - Light cannot be described entirely as either wave or particle.
  - Wave behavior is displayed in some situations, while particle behavior is displayed in others.
  - Light displays wave-particle “duality”.

Do other particles display the wave-particle duality?

- Louis de Broglie (1892-1987)

– Stated that all particles display the wave-particle duality.

– Each particle (not just photons) has a wave associated with it. The associated wave (also called De Broglie wave) satisfies two of the relations found to be true for photons:

$$p = \frac{h}{\lambda} \Leftrightarrow \lambda = \frac{h}{p}$$

$$E = hf$$

De Broglie waves and Bohr's model

- The Bohr orbits are an integer number of electron wavelengths.
- We'll prove this in the following slides

Substituting $r_n = \dfrac{n^2\hbar^2}{m_e k e^2}$ into $\mathrm{v} = \sqrt{\dfrac{k}{m_e}}\dfrac{e}{\sqrt{r}}$ yields:

$$\mathrm{v} = \sqrt{\frac{k}{m_e}}\frac{e}{\sqrt{\dfrac{n^2\hbar^2}{m_e k e^2}}} = k\frac{1}{n}\frac{e^2}{\hbar}$$

De Broglie waves and Bohr's model cont.

$$\mathrm{v} = k\frac{1}{n}\frac{e^2}{\hbar}$$

$$p = m_e \mathrm{v} = m_e k \frac{1}{n}\frac{e^2}{\hbar}$$

$$\lambda = \frac{h}{p} = \frac{h}{m_e k \frac{1}{n}\frac{e^2}{\hbar}} = \frac{nh\hbar}{m_e k e^2}$$

$$\lambda = \frac{nh\hbar}{m_e k e^2}$$

De Broglie waves and Bohr's model cont.

$$r_n = \frac{n^2\hbar^2}{m_e k e^2} \Rightarrow C_n = 2\pi r_n = 2\pi \frac{n^2\hbar^2}{m_e k e^2} = \frac{n^2 h\hbar}{m_e k e^2}$$

Circumference of orbit n

$$\lambda = \frac{nh\hbar}{m_e k e^2} \Rightarrow \frac{C_n}{\lambda} = \frac{\frac{n^2 h\hbar}{m_e k e^2}}{\frac{nh\hbar}{m_e k e^2}} = n \Rightarrow C_n = n\lambda$$

De Broglie waves and Bohr's model cont.

- The Bohr orbits are an integer number of de Broglie wavelengths
- Immediate consequence:

$$\lambda = \frac{h}{p} = \frac{h}{m\mathrm{v}}$$

$$C_n = n\lambda$$

$$C_n = 2\pi r_n$$

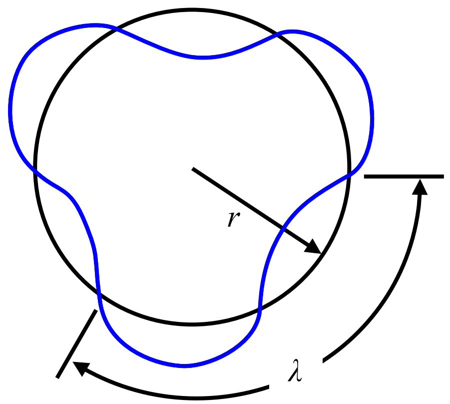
$$2\pi r_n = n\frac{h}{m\mathrm{v}}$$

$$m_e \mathrm{v} r_n = n\frac{h}{2\pi} = n\hbar$$

We have recovered Bohr's quantization of angular momentum

De Broglie waves and Bohr's model cont.

- The Bohr orbits are an integer number of de Broglie wavelengths
- We can picture a wave that goes along the Bohr orbit.



Summary

• Each particle (not just photons) has an associated wave, called De Broglie wave.

• The existence of De Broglie waves for electrons is consistent with Bohr's atomic model.

• De Broglie waves were demonstrated for other particles as well, by diffraction experiments.

• Quantum mechanics expands on these ideas (but we won't go into quantum mechanics) and shows that each system of particles has an associated wave function which describes the system's properties.

• The wave function (for any system) is found by solving the Schrödinger Equation.

Atomic and Nuclear Constituents

Atomic and Nuclear Constituents

Atom

- Electrons
- Nucleus - made up of nucleons:
  - protons
  - neutrons

The nucleus is “held together” by nuclear attraction forces. These have to be stronger than the repulsive electrostatic forces.

For neutral atoms, the number of protons in the nucleus equals the number of electrons in orbit.

Some subatomic particles

- proton
- neutron
- electron (beta particle)
- positron
- photon (gamma particle)
- neutrino
- antineutrino
- alpha particle (2 protons + 2 neutrons)

Properties (quantities) characterizing subatomic particles

- Mass (rest mass)
- charge
- spin (denoted by s)
- parity
  - property resulting from Quantum Mechanics.
  - describes the parity of the wave function

$$+ \Leftrightarrow \psi(\vec{r}) = \psi(-\vec{r})$$

$$- \Leftrightarrow \psi(\vec{r}) = -\psi(-\vec{r})$$

All these quantities are important because they are conserved in nuclear reactions.

Properties of Nuclei

- Atomic number – Z = number of protons
- Mass number – A = total number of nucleons (protons and neutrons)
- Number of neutrons – N

The atomic number Z identifies the nuclear species.

Two nuclei with the same Z but different N are called **isotopes**.

Notation: ${}^{A}_{Z}X$, where X is the chemical symbol.

Other properties of nuclei (parallel those of particles)

- Mass
- charge (+Ze)
- spin (s)
- parity

Atomic Mass Unit (amu)

- Defined as 1/12 of the mass of a ^{12}C **atom**
  That means that it is 1/12 of the ^{12}C nucleus, plus the mass of 1/2 electron.
- Atomic weight = Numerical value of the mass of an atom expressed in amu
- Molecular weight = Numerical value of the mass of a molecule expressed in amu
- 1 Mole – Quantity of a pure substance that has the same mass expressed in grams as the atom's (or molecule's) mass expressed in amu.
- 1 Mole Has $N_A = 6.023 \times 10^{23}$ atoms (molecules)
- N_A is the ratio between 1g and 1 amu.
  (There are N_A amus in a gram.)

How many Kg does an amu have?

N_A atoms of ^{12}C weigh 12 g. It follows that 1 amu weighs $1/N_A$ grams.

$$1\,\text{amu} = \frac{12(g)}{12\times N_A} = \frac{1}{N_A}(g) = \frac{1}{6.023\times 10^{23}}(g) = 1.66\times 10^{-24}(g) = 1.66\times 10^{-27}(kg)$$

Expressing mass using energy

Because of the mass-energy equivalence expressed by Einstein's formula $E = mc^2$, mass can also be expressed in units of energy over c^2.

For example:

$$1kg = \frac{1kg \times c^2}{c^2} = \frac{1 \times \left(3 \times 10^8\right)^2 (J)}{c^2} = 9 \times 10^{16} \left(\frac{J}{c^2}\right)$$

Expressing mass using energy

In nuclear physics the energy is often measured in MeV, and the mass in MeV/c^2. To find the relation between 1kg and one MeV/c^2 we write:

$$1\frac{MeV}{c^2} = \frac{10^6 eV}{\left(3\times10^8 (m/s)\right)^2} = \frac{10^6 \times 1.602\times10^{-19} C\times V}{\left(3\times10^8 (m/s)\right)^2}$$

$$= \frac{1.602\times10^{-13} J}{\left(3\times10^8 (m/s)\right)^2} = 1.78\times10^{-30} Kg$$

Instead of saying that the mass if a particle is X MeV/c^2 it is customary to just say that the mass is x MeV. What is really meant is that the total energy of that particle is X MeV, and hence its mass is X MeV/c^2. One just omits mentioning c^2.

MeV Equivalent of 1 amu

$$E_{amu} = \frac{1}{N_A} 1g \times c^2 = \frac{1}{6.023 \times 10^{23}} \times 10^{-3} Kg \times \left(3 \times 10^8 \frac{m}{s} \right)^2 =$$

$$\frac{1}{6.023 \times 10^{23}} \times 10^{-3} \times \left(3 \times 10^8\right)^2 J =$$

$$1.494 \times 10^{-7} J \frac{J}{MeV} MeV = .494 \times 10^{-7} \frac{C \times V}{10^6 e \times V} MeV =$$

$$1.494 \times 10^{-7} \frac{C \times V}{10^6 \times 1.6 \times 10^{-19} C \times V} MeV = 933 MeV$$

Examples of elementary particle mass

particle	mass		
	kg	amu	MeV/c^2
proton	1.6726E-27	1.007276	938.28
neutron	1.6750E-27	1.008665	939.57
electron	9.1090E-31	5.486E-4	0.511

Atomic Weight of a Mixture of Atoms Expressed by Atomic Fraction

Consider a mixture of 30% (by atom) C and 70% (by atom) Al. What is the average atomic weight of the mixture?

<u>Answer</u>

- Assume there are N atoms in total
- of these
  - N_C=0.3N are C
  - N_{Al}=0.7N are Al

Atomic Weight of a Mixture of Atoms Expressed by Atomic Fraction

The total mass of the mixture (in amu) is:

$$m = N_C M_C + N_{Al} M_{Al} = 0.3NM_C + 0.7NM_{Al}\,(amu)$$

The average mass of one atom (in amu) is:

$$\overline{M} = \frac{m}{N} = \frac{0.3NM_C + 0.7NM_{Al}}{N} =$$

$$= 0.3M_C + 0.7M_{Al} = 0.3 \times 12 + 0.7 \times 13 = 12.7\,(amu)$$

Atomic Weight of a Mixture of Atoms Expressed by Atomic Fraction

In general

For a mixture of n types of atoms, each with atomic fraction $X_i = N_i/N$, the average atomic weight is:

$$M = \sum_{i=1}^{n} X_i M_i$$

If the different types of atoms are isotopes of the same atom, the atomic fractions are called *isotopic abundances*.

Atomic Weight of a Mixture of Atoms Expressed by Weight (Mass) Fraction

In general

For a mixture of n types of atoms, each with mass fraction $Y_i = m_i/m$, the average atomic weight is:

$$M = \frac{1}{\sum_{i=1}^{n} \frac{Y_i}{M_i}}$$

Enrichment

For a mixture of ^{235}U and ^{238}U, enrichment, r, is defined as the fraction of ^{235}U.

Enrichment can be expressed by atom or by weight (mass) fraction.

Usually expressed as percent.

$$r(a/o) = \frac{N_{235}}{N_{235} + N_{238}} \times 100$$

$$r(w/o) = \frac{m_{235}}{m_{235} + m_{238}} \times 100$$

Properties and Structure of Nuclei

Nuclear Radius

Assume that nuclei are made of "nuclear material" of the same density ρ for all species of nuclei.

It follows that the mass of the nucleus is given by:

$$m = \rho V = \rho \frac{4\pi R^3}{3}$$

Nuclear Radius

The mass of the nucleus is given also by the mass of its constituents (neutrons and protons)

$$m = Nm_n + Zm_p$$

Because the mass of the proton and the one of the neutron are almost equal to 1 amu, we can write:

$$m = Nm_n + Zm_p \cong N\,amu + Z\,amu = (N + Z)\,amu = A\,amu$$

Nuclear Radius

By writing the equality between the two masses, we have:

$$A\,amu = \rho \frac{4\pi R^3}{3}$$

Solving for R^3, we obtain

$$R^3 = A\left(\frac{3}{4\pi\rho}amu\right)$$

Nuclear Radius

Solving for R, by taking the cube root on both sides, we have:

$$R = \sqrt[3]{A}\sqrt[3]{\frac{3}{4\pi\rho}amu}$$

It turns out that:

$$\sqrt[3]{\frac{3}{4\pi\rho}amu} = 1.25\times10^{-15}m$$

So:

$$R = 1.25\times10^{-15}\times\sqrt[3]{A}\ \text{meters}$$

Binding Energy

Since particles that constitute the nucleus stay together (held by nuclear interaction forces), the total (rest) energy of the nucleus must be lower than the total (rest) energy of the particles if they were separated.

$$B = \left[\left(A - Z \right) \times E_{0\,neutron} + Z \times E_{0\,proton} \right] - E_0 \left({}^{A}_{Z}X \right)$$

This is called the **Binding Energy**

In the above, E_0 denotes rest energy.

Binding Energy

Einstein's energy formula translates into:

(masses are assumed to be rest masses)

$$E_{0\,neutron} = m_{neutron} c^2$$

$$E_{0\,proton} = m_{proton} c^2$$

$$E\left({}^{A}_{Z}X\right) = M\left({}^{A}_{Z}X\right)c^2$$ (M is rest mass of the nucleus.)

$$B = \left[(A-Z)\times m_{neutron}c^2 + Z\times m_{proton}c^2\right] - M\left({}^{A}_{Z}X\right)c^2$$

$$B = c^2\left\{\left[(A-Z)\times m_{neutron} + Z\times m_{proton}\right] - M\left({}^{A}_{Z}X\right)\right\} = c^2\Delta$$

The mass of the nucleus is smaller than the sum of the masses of its constituents

The difference, Δ, is called the ***mass defect***

Alternate expression for the mass defect

Using the nuclear mass to calculate the mass defect can be difficult because, most of the time, what is given in tables is the mass of neutral atoms, rather than the mass of their nuclei.

To use the atomic masses instead of the nuclear masses, we can add and subtract the mass of electrons. (We will also ignore the binding energy of the electrons. However, that energy is much smaller that the nuclear binding energy, so we can safely neglect it.) Hence:

$$\Delta = \left[(A - Z) \times m_{neutron} + Z \times m_{proton} \right] - M \left({}^{A}_{Z}X \right) =$$

$$\left[(A - Z) \times m_{neutron} + Z \times \left(m_{proton} + m_{e} \right) \right] - \left(M \left({}^{A}_{Z}X \right) + Z \times m_{e} \right) =$$

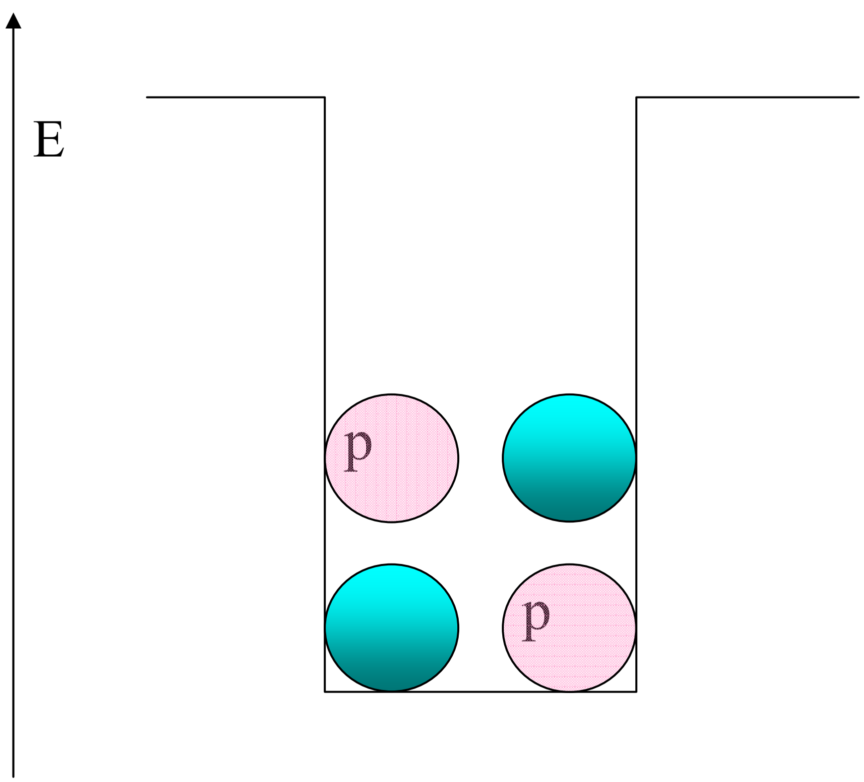
$$(A - Z) \times m_{neutron} + Z \times \mathrm{M} \left({}^{1}_{1}H \right) - \mathrm{M} \left({}^{A}_{Z}X \right)$$

Where $\mathrm{M}\left({}^{A}_{Z}X \right)$ is the rest **atomic** mass of element ${}^{A}_{Z}X$

Nuclear Models

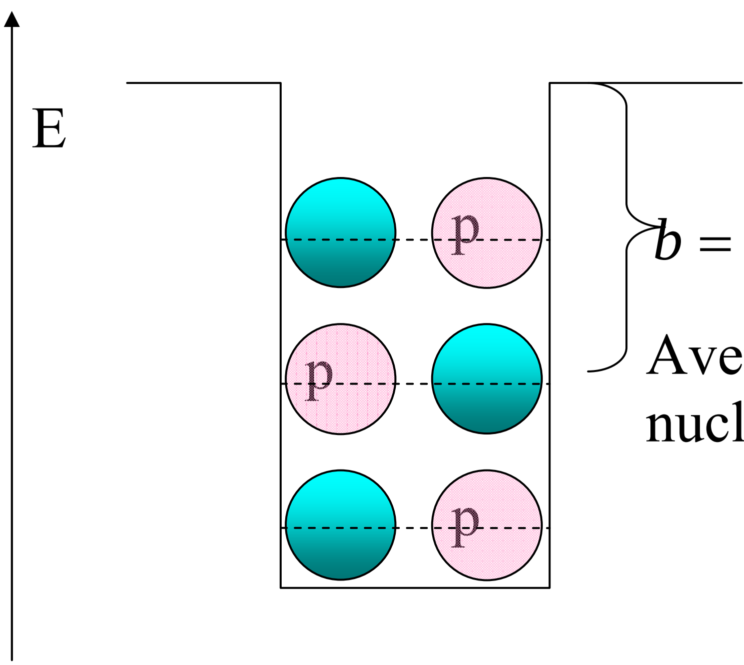
Shell Model - Potential Well

- We can picture the nucleons (protons and neutrons) as "living" in a "potential well" created by the nuclear forces.
- The binding energy is the energy that needs to be communicated to the nucleons to allow all of them to exit the well.



More details on the potential well

- Nucleons can occupy different energy levels in the well, just like electrons can occupy different energy levels in an atom.
- The state of the nucleus is given by the states (energy, spin, parity) of all its nucleons.
- Pauli's exclusion principle applies (No two nucleons can occupy the same state).

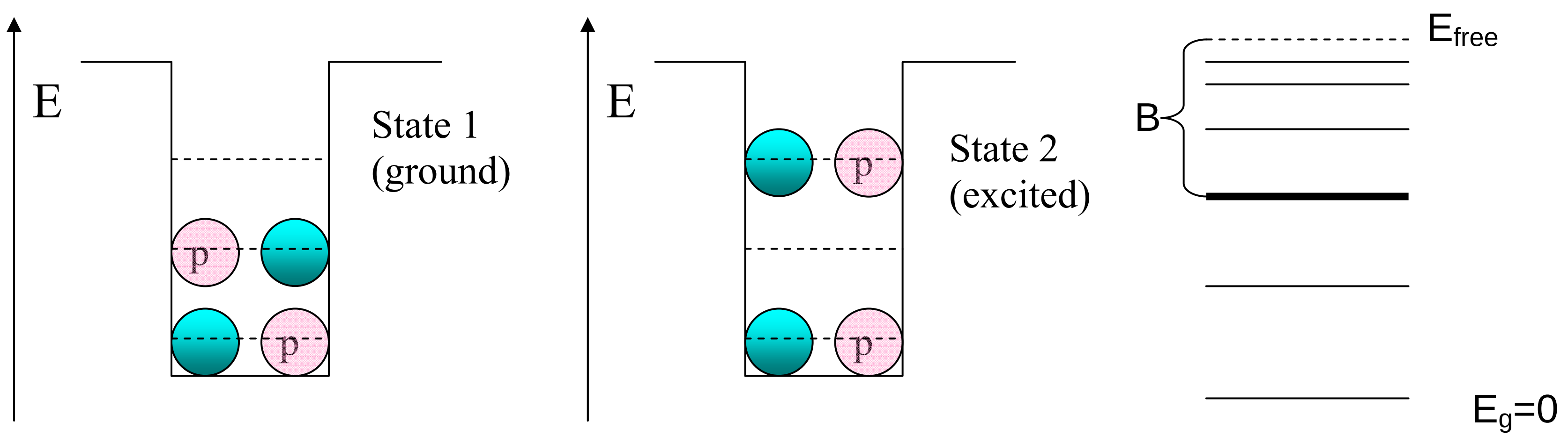


$$b = \frac{B}{A}$$

Average binding energy per nucleon

More details on the potential well

- Depending on the "arrangement" of nucleons on energy levels inside the well, the nucleus can have different binding energies.
- The lowest energy level of the nucleus (corresponding to the largest binding energy) is called the **ground level**, and the corresponding state is called the **ground state**.
- Higher energy levels are called **excited levels**, and the corresponding states are called **excited states**.

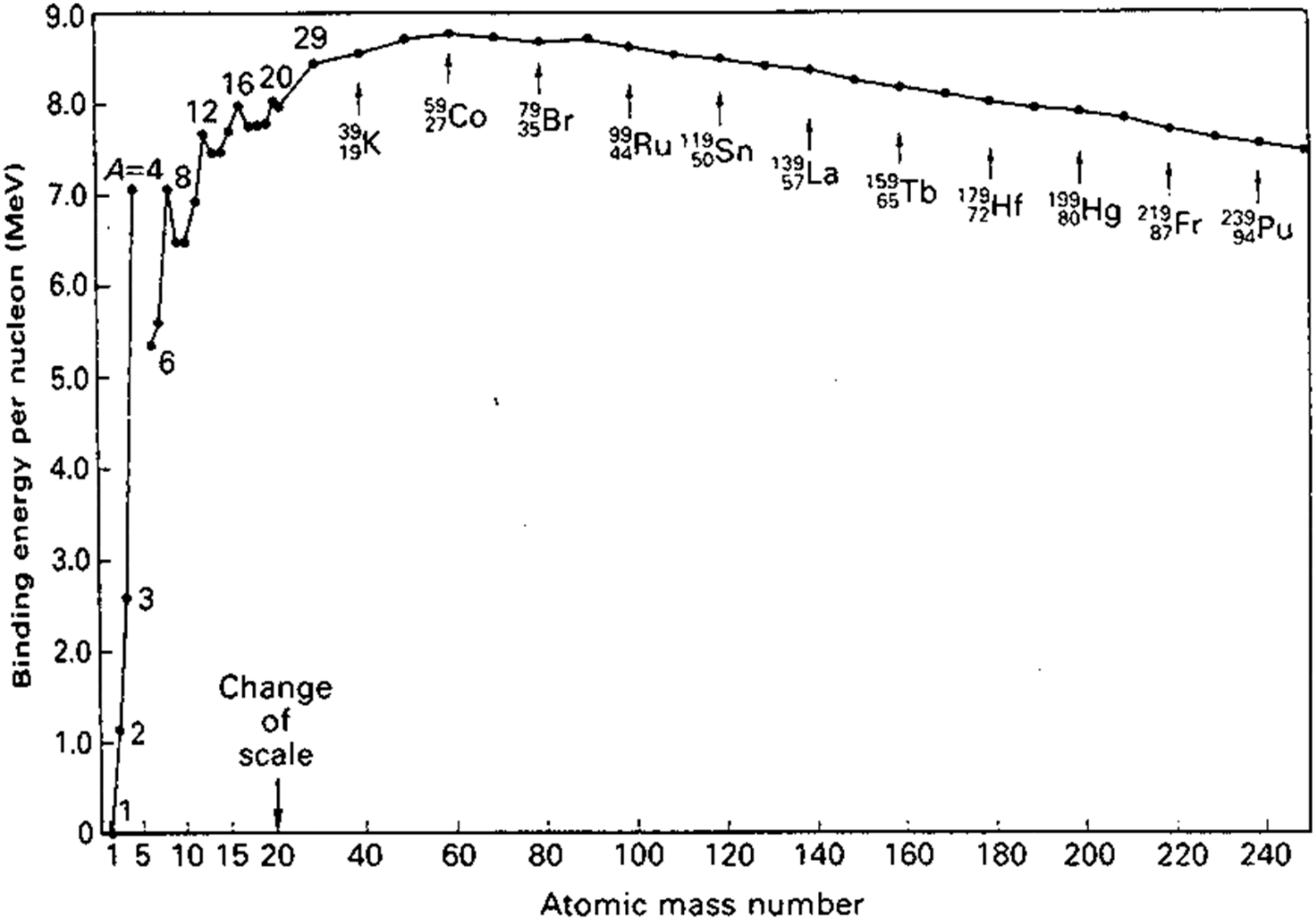


$$E_{excited} > E_{ground}$$

$$B_{ground} > B_{excited}$$

$$M\left({}_Z^N X^{excited}\right) > M\left({}_Z^N X^{ground}\right)$$

Binding Energy per Nucleon



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Liquid Drop Nuclear Model

- Attempts to express the binding energy as a function of nuclear characteristics.
- Leads to a semiempirical formula.
  - Shape of formula determined from the model
  - Values of constants determined from measurement

$$B = a_v A - a_s A^{\frac{2}{3}} - a_c \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A^2} + \delta(Z, A)$$

Liquid Drop Model – Meaning of Terms

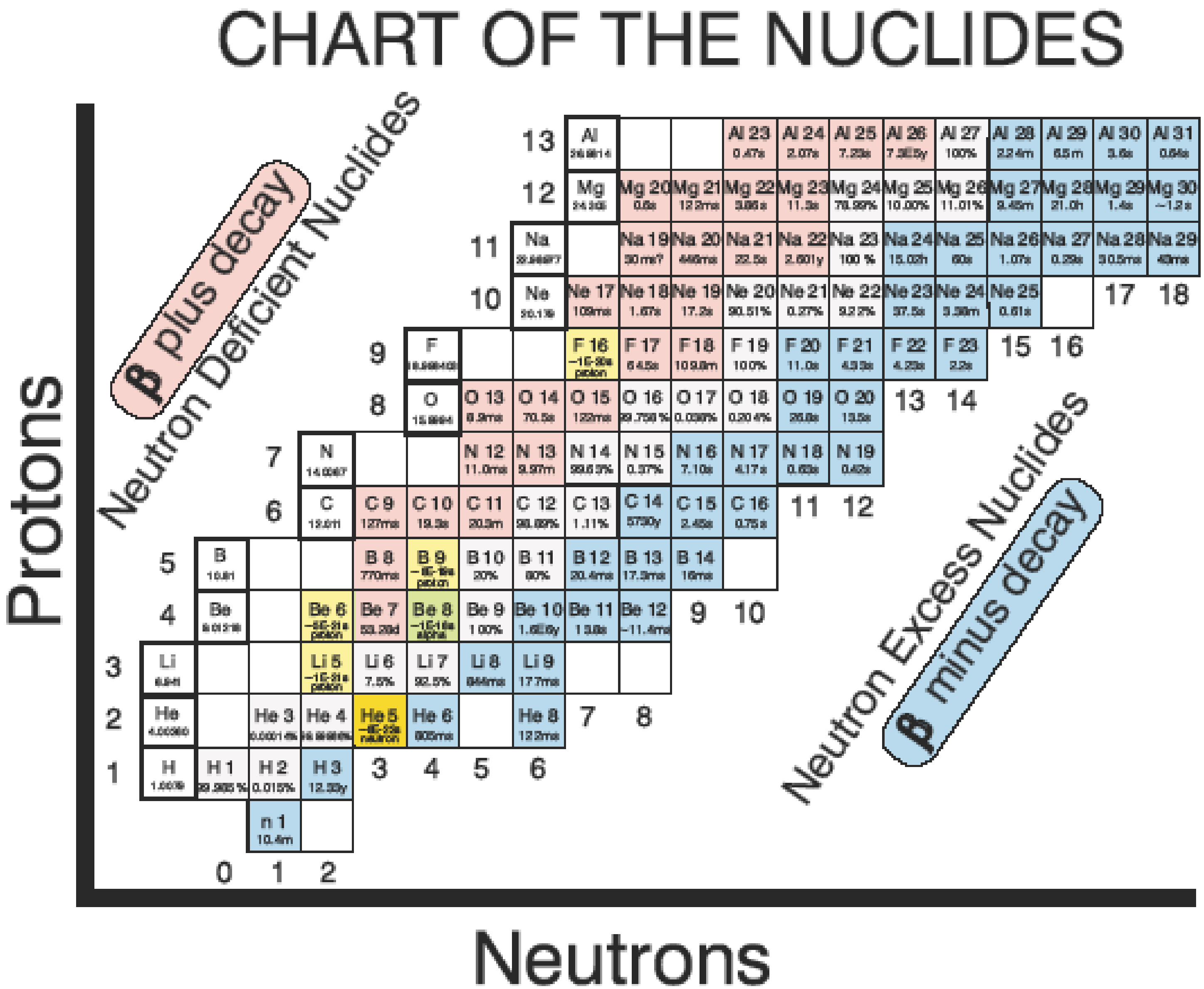
- **a_v** – Volume effect – proportional to the "volume" of the nucleus, which can be considered to be roughly proportional to A. This term was introduced because it was observed that the binding energy per nucleon is almost constant.
- **a_s** – Surface effect – proportional to the "surface" of the nucleus, roughly proportional to $A^{2/3}$. This negative term was introduced because the nucleons situated close to the surface have fewer neighbors, and hence contribute less to the binding energy.
- **a_c** – Coulomb effect – electrostatic repulsion between protons has a potential energy $\frac{Z(Z-1)e^2}{r} \propto \frac{Z(Z-1)}{A^{\frac{1}{3}}}$
- **a_A** – Asymmetry effect. It vas observed that nuclei with N=Z are more stable, hence the binding energy is probably smaller if Z and N differ. This term accounts for that effect.
- $\delta(Z, A)$ - Pairing term. Introduced because it was found experimentally that two protons or two neutrons are bound stronger than a proton and a neutron. It is zero for odd A, $-a_p \frac{1}{A^{\frac{1}{2}}}$ for both Z and N odd and $+a_p \frac{1}{A^{\frac{1}{2}}}$ for both Z and N even.

Liquid Drop Model Numerical values of coefficients

a_V	15.7 MeV
a_S	17.8 MeV
a_C	0.71 MeV
a_A	23.6 MeV
a_P	12.0 MeV

Radioactivity

Nuclear Stability



Radioactivity

- Discovered first by Henri Becquerel (1852-1908).
- Becquerel discovered that a mineral containing Uranium would darken a photographic plate even when the latter was wrapped in opaque paper.
- In 1903 Becquerel shared the Physics Nobel Prize with Pierre and Marie Curie, for their discovery and work on radioactivity.

Radioactive Decay

- Some nuclei are **stable**, while others are **unstable**.
- Unstable nuclei **decay**, by emitting a particle and changing into a different nucleus.
- Most common types of decay (others possible too):
  - Alpha ($^{4}_{2}\alpha$), Helium nucleus emission
  - Beta ($^{0}_{-1}\beta$), electron emission
  - Beta plus ($^{0}_{1}\beta$) positron emission
  - Gamma ($^{0}_{0}\gamma$), photon emission (no change in nuclear species)
  - Electron capture (an electron is "captured" rather than emitted)

Radioactive Decay

$$ {}_{Z}^{A}X \rightarrow {}_{Z-n}^{A-m}Y + {}_{n}^{m}P $$

(Parent Nucleus → Daughter Nucleus + Emitted Particle)

- Charge and number of nucleons are conserved.
- For gamma decay, technically the nucleus does not change into a different one. Only its energy state changes.
- Electron capture (still classified as "decay")

$$ {}_{Z}^{A}X + {}_{-1}^{0}e \rightarrow {}_{Z-1}^{A}Y $$

Alternative Notation (no chemical symbol)

- General decay

$$(Z, A) \rightarrow (Z - n, A - m) + (n, m)$$

- Alpha

$$(Z, A) \rightarrow (Z - 2, A - 4) + {}_{2}^{4}\alpha$$

- Beta minus

$$(Z, A) \rightarrow (Z + 1, A) + {}_{-1}^{0}\beta + \widetilde{\upsilon}$$

- Electron capture

$$(Z, A) + e^{-} \rightarrow (Z - 1, A)$$

$$(Z, A) + {}_{-1}^{0}\beta \rightarrow (Z - 1, A)$$

Characteristics of Radioactive Decay

- Nuclei decay randomly.
  - It is impossible to predict which nuclei will decay in a given period of time, and which not.
  - It is impossible to predict when a particular nucleus will decay.
- On average, for large initial numbers of nuclei and for short periods of time Δt, the number of nuclei that decay within Δt is proportional to the time Δt, and to the original number of nuclei present at the beginning of the time interval.

Derivation of the Law of Radioactive Decay

- Let N(t) be the number of X-type nuclei present at time t.
- Let Δt be a short time interval.
- According to the second bullet on the previous slide, we have, on average:

$$\Delta N = -N(t) + N(t+\Delta t) = -\lambda \times N(t) \times \Delta t \qquad (1)$$

- λ is called the decay constant, is dependent on nucleus type, and is measured in s^{-1}.

Derivation of the Law of Radioactive Decay

- The previous can be rewritten as:

$$\frac{\Delta N}{\Delta t} = -\lambda N(t) \qquad (2)$$

- which, considering that Δt is small, yields:

$$\frac{dN}{dt} = -\lambda N(t) \qquad (3)$$

Derivation of the Law of Radioactive Decay

Eq. (3) is an ordinary differential equation with constant coefficients. Its solution is of the form:

$$e^{-\lambda t+c} \equiv Ce^{-\lambda t} \; ; \qquad C = e^{c}$$

The multiplicative constant C can be determined from the number of nuclei present at t=0.

$$N(0) \equiv N_0 = C \times e^{-\lambda \times 0} = C$$

It follows that the number of X-type nuclei is given at any time t by:

$$N(t) = N_0 \times e^{-\lambda t}$$

Law of Radioactive Decay

Example

- At t=0, a sample of ^{24}Na weights 1.0 mg. How many beta particles are emitted in an hour? ($\lambda = 1.2836 \times 10^{-5}\ s^{-1}$)
- Solution
  - The number of emitted particles equals the number of decayed nuclei:

$$\Delta N = N_0 - N(t) = N_0 - N_0 \times e^{-\lambda t} = N_0 \times \left(1 - e^{-\lambda t}\right)$$

- The initial number of Na nuclei is:

$$N_0 = \frac{m}{M} = \frac{1.0 \times 10^{-3} g}{24 amu} = \frac{1.0 \times 10^{-3}}{24} \times \frac{g}{amu} =$$

$$\frac{1.0 \times 10^{-3}}{24} \times N_A = \frac{1.0 \times 10^{-3}}{24} \times 6.023 \times 10^{23} = 2.51 \times 10^{19}$$

- Hence the number of emitted particles is:

$$\Delta N = 2.51 \times 10^{19} \times \left(1 - e^{-1.2836 \times 10^{-5} \times 3600}\right) = 1.133 \times 10^{18}$$

Half Life

- Definition
  - The half life, $T_{1/2}$, of a radioactive species is the time after which the initial number of nuclei decreases to one half.
- Expression
  - By definition:

$$N(T_{1/2}) = \frac{N_0}{2}$$

Expression of Half-Life

- The definition of half life is equivalent to:

$$N_0 \times e^{-\lambda \times T_{1/2}} = \frac{N_0}{2}$$

- Dividing be N_0 we obtain:

$$e^{-\lambda \times T_{1/2}} = \frac{1}{2}$$

- By taking the natural logarithm of both sides we get:

$$-\lambda \times T_{1/2} = \ln\left(\frac{1}{2}\right) = -\ln(2)$$

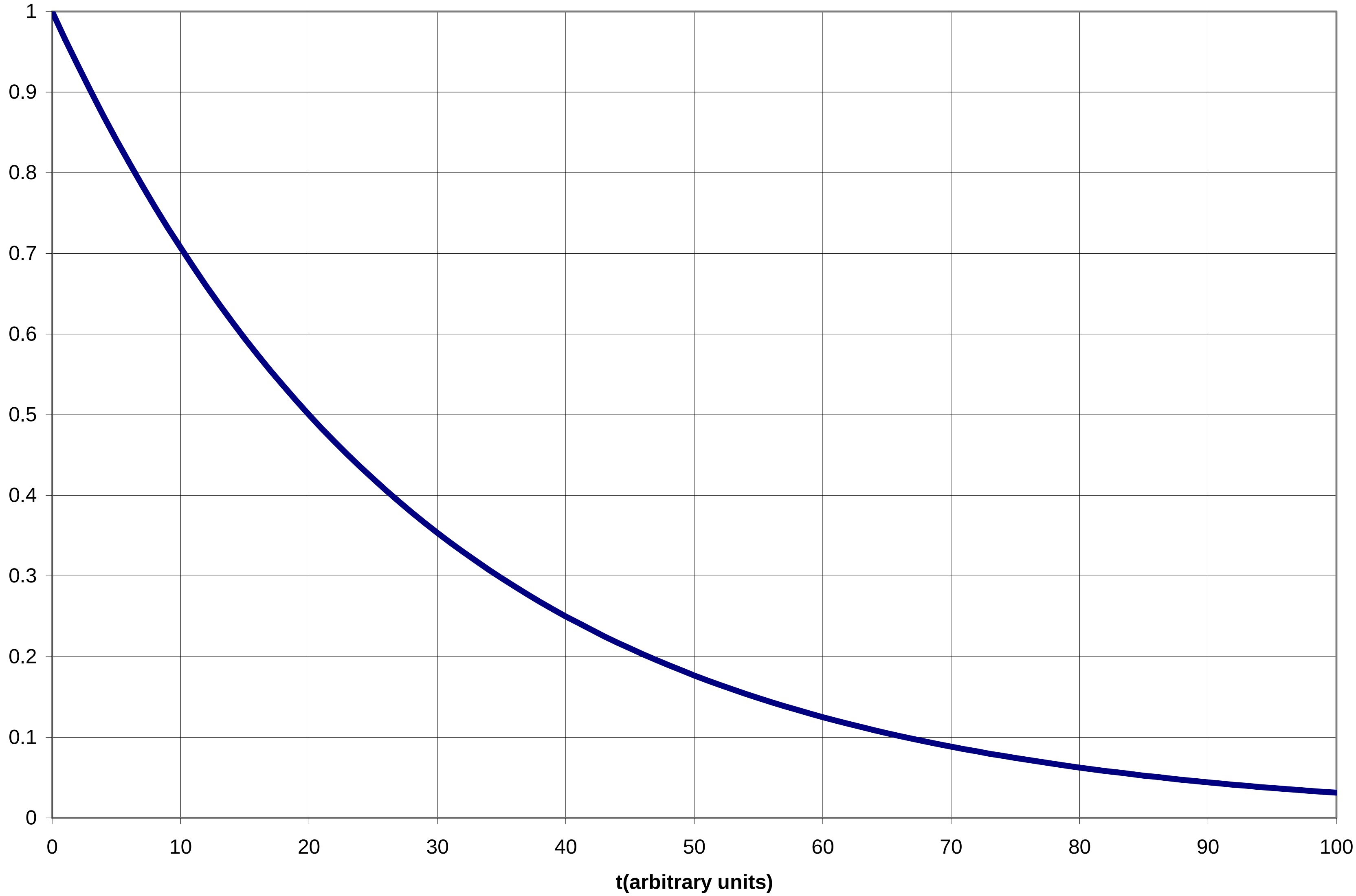
- Finally, we can solve for $T_{1/2}$:

$$T_{1/2} = \frac{\ln(2)}{\lambda}$$

Radioactive Decay and Half Life Important Notes

- Half life can be measured from any moment of time. The number of nuclei left after $T_{1/2}$ elapses will be half of those existent at t_0.
- According to the radioactive decay law, the number of parent nuclei keeps halving every $T_{1/2}$, but never reaches zero. However, it can become negligibly small.
- As the number of remaining nuclei becomes small, deviations from the law of radioactive decay start to appear, as the law of radioactive decay is valid ***on average***.
  We cannot have 2.5 parent nuclei left. What such a number means is that we can have 2 or maybe 3 nuclei left in different experiments, such that the average is 2.5

Exponential Decay



Law of Radioactive Decay – Probabilistic Interpretation

- $N(t)$ out of N_0 nuclei do not decay.
- It cannot be determined *a priori* which nuclei do not decay and which do.
- The ratio $N(t)/N_0$ can be interpreted as the probability of one nucleus **not decaying** after time t.

$$\frac{N(t)}{N_0} = P_{ND} = e^{-\lambda \times t}$$

- Conversely, the probability that a nucleus **does decay** after time t is:

$$P_D = 1 - P_{ND} = 1 - e^{-\lambda \times t} \neq e^{-\lambda \times t}$$

Activity

- The rate at which a radioactive sample decays is called **activity.**

$$\Lambda(t) \equiv -\frac{dN(t)}{dt}$$

- Equivalent definition

$$\Lambda(t) = -\lambda N(t) \Leftarrow -\frac{d}{dt} N_0 e^{-\lambda t} = \lambda N_0 e^{-\lambda t} = \lambda N(t)$$

- Units:
  - 1 decay/second = 1 Becquerel (Bq)
  - 1 Curie = $3.7\text{x}10^{10}$ Bq

Average Life of a Nucleus

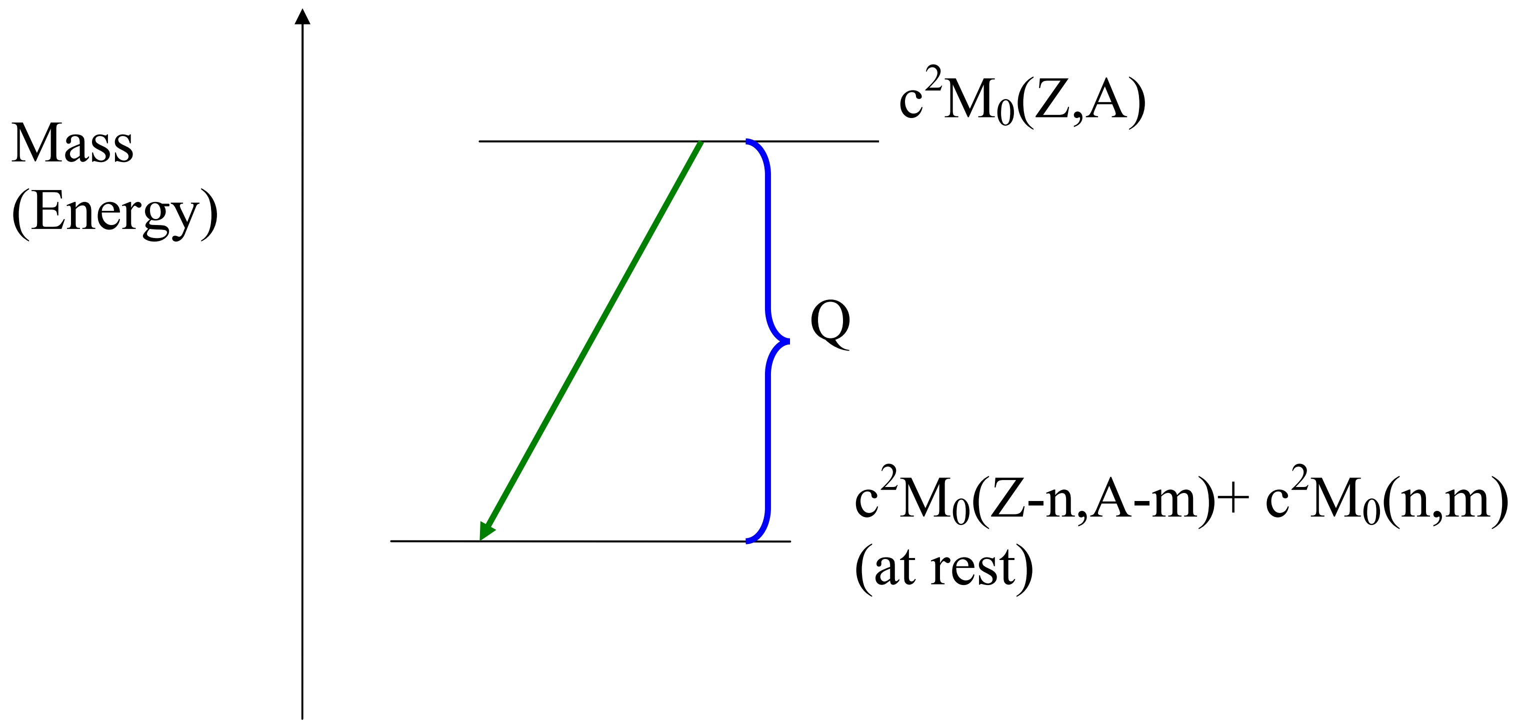
- At t=0 there are N_0 parent nuclei.
- At time *t*, there are N parent nuclei left.
- At time *t*, $\Lambda(t)dt = \lambda N_0 e^{-\lambda t} dt$ decay in *dt*
- These nuclei have "lived" *t* before decaying.
- To get the average life, we need to sum (integrate) over *dt* and divide by the initial number of nuclei.

$$\tau = \frac{\int_0^\infty t\Lambda(t)dt}{N_0} = \frac{\int_0^\infty t\lambda N_0 e^{-\lambda t} dt}{N_0}$$

$$= \frac{N_0 \lambda \int_0^\infty t e^{-\lambda t} dt}{N_0} = \lambda \int_0^\infty t e^{-\lambda t} dt =$$

$$\lambda \left[\left(-\frac{1}{\lambda} t e^{-\lambda t} \right) \Big|_0^\infty + \int_0^\infty \frac{1}{\lambda} e^{-\lambda t} dt \right] = \int_0^\infty e^{-\lambda t} dt = -\frac{1}{\lambda} \left(e^{-\lambda t} \right) \Big|_0^\infty = \frac{1}{\lambda}$$

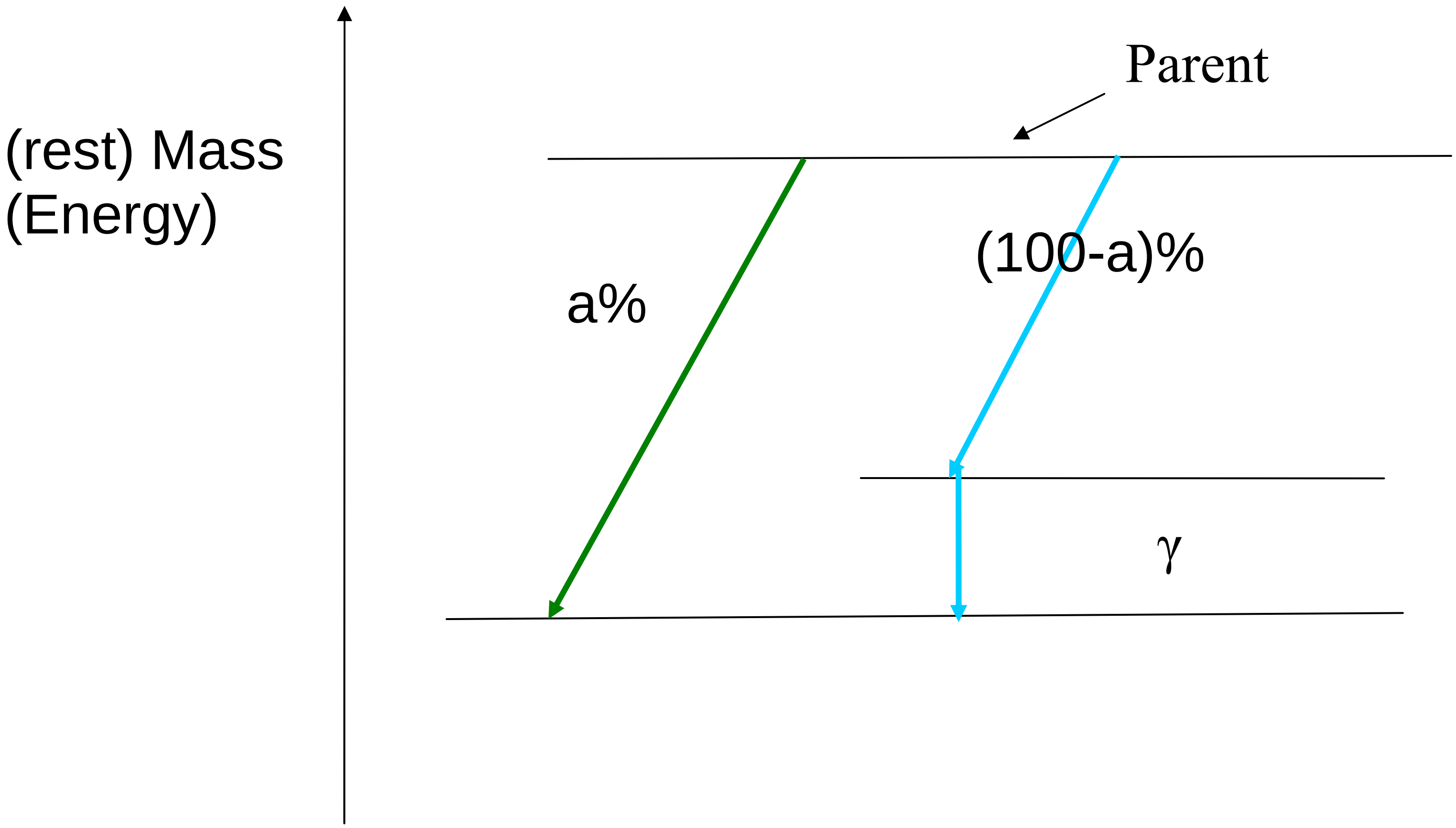
Energy-Level Diagrams for Decay and Decay Scheme



- $Q=[M_0(Z,A)-M_0(Z-n,A-m)-M_0(n,m)]c^2$
- $Q>0$ in order for the decay to be energetically possible
- By convention, the lowest energy on this diagram is taken to be zero (Energy is expressed relative to the lowest value.).

Multimodal Decay

- Some nuclei can decay in more than one way



Branching factors

- Fraction of nuclei that decay in a certain mode
- Have to add up to 1. (100%)

- Consider a species of nucleus that can decay by either mode 1, or mode 2.
- Let dN_1 be the number of nuclei that decay by mode 1 in dt
- Let dN_2 be the number of nuclei that decay by mode 2 in dt
- Let dN be the total number of nuclei that decay in dt.

$$f_1 = \frac{dN_1}{dN}$$

$$f_2 = \frac{dN_2}{dN}$$

$$f_1 + f_2 = 1$$

Branching factors and derived quantities

- Partial decay constants

$$\lambda = \frac{dN}{dt \times N}$$

$$\lambda_1 = \frac{dN_1}{dt \times N} = \frac{dN_1}{dN}\frac{dN}{dt \times N} = f_1 \lambda$$

$$\lambda_2 = \frac{dN_2}{dt \times N} = \frac{dN_2}{dN}\frac{dN}{dt \times N} = f_2 \lambda$$

$$\lambda_1 + \lambda_2 = \lambda$$

- Partial half-lives (What the half life would be if only that decay mode was present).

$$T_1 = \frac{\ln 2}{\lambda_1}$$

$$T_2 = \frac{\ln 2}{\lambda_2}$$

$$\frac{1}{T_1} + \frac{1}{T_2} = \frac{1}{T}$$

Decay Chains
(Radioactive Families)

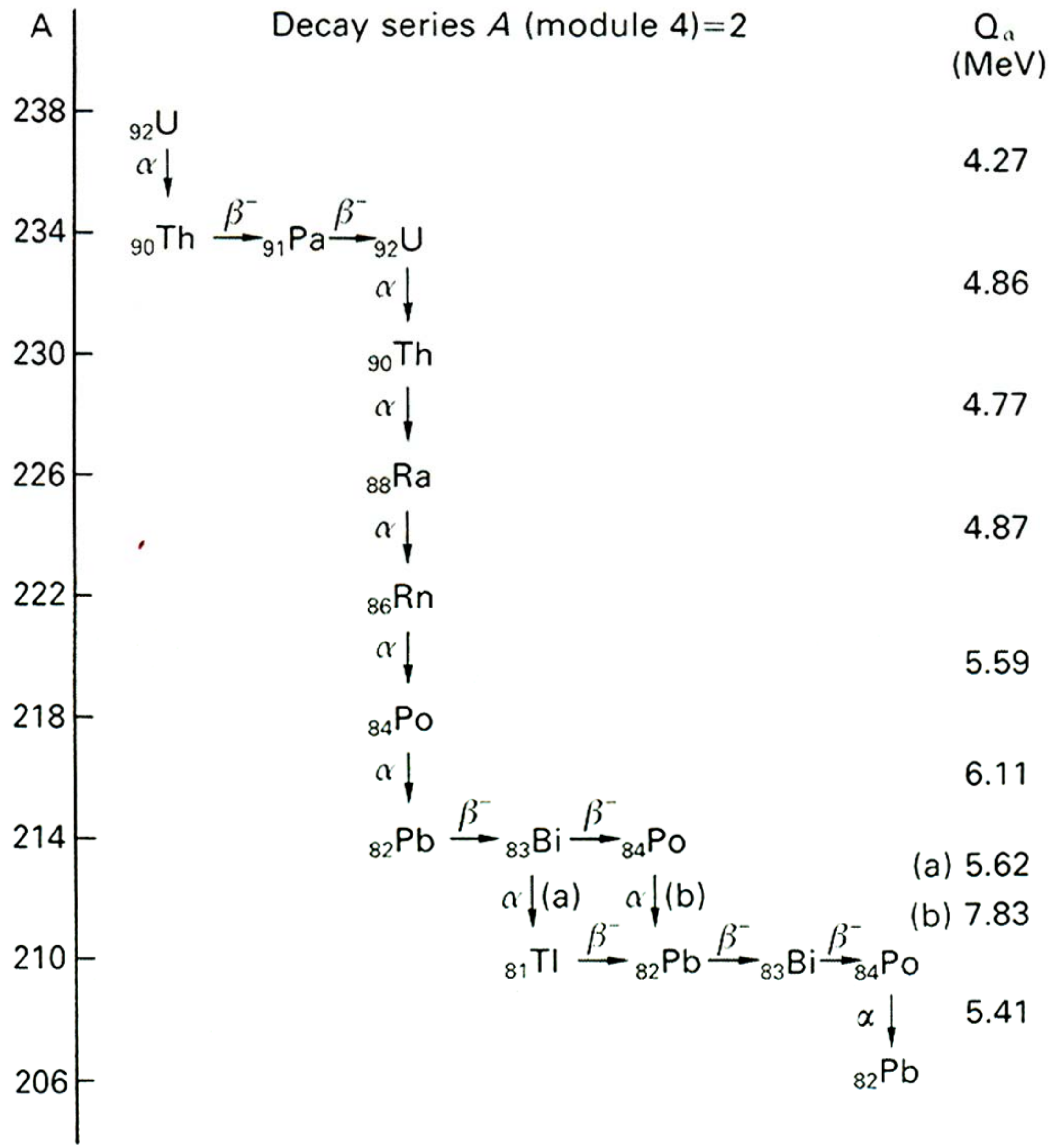
- Consider a nuclide whose daughter is also unstable and decays.

$$ {}^{A}_{Z}Q \rightarrow {}^{A-m}_{Z-n}R + {}^{m}_{n}P $$

$$ {}^{A-m}_{Z-n}R \rightarrow {}^{A-m-m'}_{Z-n-n'}S + {}^{m'}_{n'}P' $$

- This is called a decay chain.
- Chains can have more than two members. We then talk about radioactive families or series.

Example of Radioactive Family



(Reproduced from W.S.C. Williams "Nuclear and Particle Physics")

Nuclear Reactions

Nuclear Reactions

General Expression

$$ {}^{A_{x1}}_{Z_{1x}}X_1 + {}^{A_{x2}}_{Z_{x2}}X_2 \rightarrow {}^{A_{y1}}_{Z_{y1}}Y_1 + {}^{A_{y2}}_{Z_{y2}}Y_2 $$

Q value

$$ Q = \left[M_0\left({}^{A_{x1}}_{Z_{1x}}X_1 \right) + M_0\left({}^{A_{x2}}_{Z_{x2}}X_2 \right) - M_0\left({}^{A_{y1}}_{Z_{y1}}Y_1 \right) - M_0\left({}^{A_{y2}}_{Z_{y2}}Y_2 \right) \right] c^2 $$

M_0 are rest masses of nuclei/particles

- $Q>0$ – exothermic reaction (provides energy to the outside)
- $Q<0$ – endothermic reaction (needs energy from outside in order to proceed)

Conservation Laws

- The following quantities are conserved in a nuclear reaction
  - charge
  - number of nucleons
  - energy
  - momentum

Conservation Laws

- Conservation of charge

$$Z_{x1} + Z_{x2} = Z_{y1} + Z_{y2}$$

- Conservation of number of nucleons

$$A_{x1} + A_{x2} = A_{y1} + A_{y2}$$

Conservation Laws

If additional particles enter or exit the reaction, their charge, number of nucleons, energy and momentum need to be accounted for when writing the conservation laws

Example

$$^{A_{x1}}_{Z_{1x}}X_1 + {}^{A_{x2}}_{Z_{x2}}X_2 \rightarrow {}^{A_{y1}}_{Z_{y1}}Y_1 + {}^{A_{y2}}_{Z_{y2}}Y_2 + e^-$$

Conservation of charge

$$Z_{x1} + Z_{x2} = Z_{y1} + Z_{y2} - 1$$

We can represent the electron as $^{0}_{-1}e$

Conservation Laws

Conservation of momentum

$$\vec{P}(X_1) + \vec{P}(X_2) = \vec{P}(Y) + \vec{P}(Y_2)$$

Conservation of energy

$$\left[M\left({}^{A_{x1}}_{Z_{1x}} X_1 \right) + M\left({}^{A_{x2}}_{Z_{x2}} X_2 \right) \right] c^2 = \left[M\left({}^{A_{y1}}_{Z_{y1}} Y \right) + M\left({}^{A_{y2}}_{Z_{y2}} Y_2 \right) \right] c^2$$

Note: M is the (relativistic) total mass

Conservation Laws

Accounting for the definition of the Q value, we also have:

$$\left[M_0\left({}^{A_{x1}}_{Z_{1x}}X_1\right)+M_0\left({}^{A_{x2}}_{Z_{x2}}X_2\right)\right]c^2=\left[M_0\left({}^{A_{y1}}_{Z_{y1}}Y\right)+M_0\left({}^{A_{y2}}_{Z_{y2}}Y_2\right)\right]c^2+Q$$

– The liberated energy (Q) is equal to the difference between the total kinetic energy of the products [including all emitted particles (photons or other)] and the total kinetic energy of the reactants

Interaction of Radiation with Matter

Atom Density

- Also called ***number density***.
- Is the Number of Atoms per Unit Volume
- Connection with (mass) density
  - n = # of atoms in volume V
  - M = atomic mass of each atom
  - N = Atom density

$$N = \frac{n}{V}$$

$$\rho = \frac{m}{V}$$

$$\Rightarrow \quad \rho = \frac{m}{V} = \frac{nM}{V} = M\,\frac{n}{V} = MN$$

Mechanisms of Interaction for Charged Particles

Heavy Charged Particles e.g. alpha particles

- Interact mostly with electrons (there are usually much more electrons than nuclei) via Coulombic force
- Are much heavier than electrons
- Lose little energy in each individual interaction with any one electron
- Eventually do slow down as a consequence of the many interactions
- Have straight-line trajectories
- Electrons are knocked out of their orbits and atoms become ionized (Hence the name “ionizing radiation”)
- Behave like bowling bowls in a space filled with golf balls

Linear Stopping Power

$$S = -\frac{dE}{dx}$$

- Bethe's formula

$$S = \frac{4\pi e^4 z^2}{m_0 \mathrm{v}^2} NB$$

$$B \equiv Z\left[\ln\frac{m_0 \mathrm{v}^2}{I} - \ln\left(1 - \frac{\mathrm{v}^2}{c^2}\right) - \frac{\mathrm{v}^2}{c^2}\right]$$

- Range (tens of microns)

Fast Light Charged Particles (electrons)

- Interact mostly with electrons (there are usually much more electrons than nuclei) via Coulombic force
- Are much of the same mass as electrons
- Can lose a lot of energy in each individual interaction with any one electron
- Slow down quickly, after only few collisions.
- Have broken-line trajectories
- Can be backscattered
- Atomic electrons are knocked out of their orbits and atoms become ionized (Hence the name “ionizing radiation”)
- When accelerated, incident electrons produce bremsstrahlung (electromagnetic radiation – photons)
- Trajectory of an electron is a broken line (possible backscatter)
- Range (millimeters)

Mechanisms of Interactions for Neutral Particles

Photons

Can have several types of interactions (all depend on energy)

- Photoelectric effect
- Compton Scattering
- Pair production
  - A highly energetic (E>1.02MeV) photon is stopped (by collision with a heavy nucleus) and its energy is converted into an electron and a positron emitted in opposite directions

$$\gamma \rightarrow e^- + e^+$$

Neutrons

- Interact with nuclei via nuclear forces, since they have no charge, hence they cannot interact electrostatically with electrons
- Possible reactions
  - Elastic scattering
  - Inelastic Scattering
  - Absorption
    - radiative capture
    - fission
  - (n, 2n)

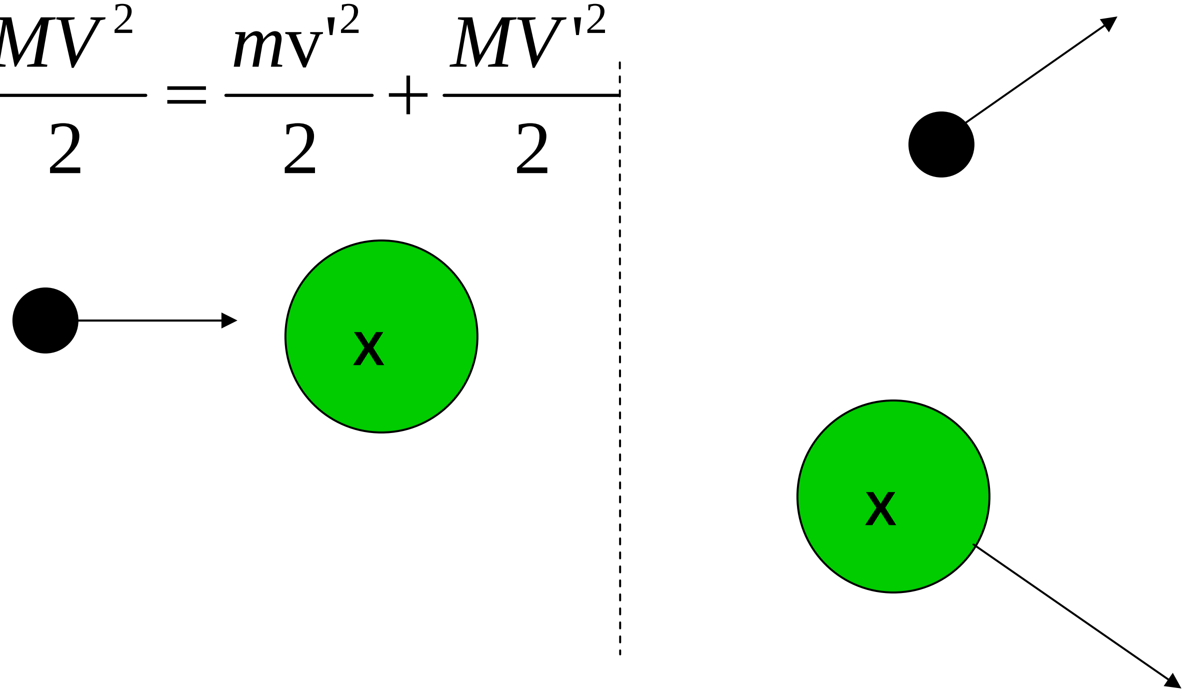
Neutron Elastic Scattering

$$n + {}^{A}_{Z}X \rightarrow n + {}^{A}_{Z}X$$

Kinetic energy is conserved

$$KE_n + KE_X = KE'_n + KE'_X$$

$$\frac{m\text{v}^2}{2} + \frac{MV^2}{2} = \frac{m\text{v}'^2}{2} + \frac{MV'^2}{2}$$



The incident neutron is slowed down by elastic scattering

Some of its kinetic energy is transferred to the target nucleus

Energy Loss in Elastic Scattering Collisions - Moderation

$$\overline{E}' = \frac{1}{2}\left[1 + \left(\frac{A-1}{A+1}\right)^2\right]E = \frac{1+\alpha}{2}E$$

Scattering of heavy nucleus (^{235}U)- small energy loss (poor moderator)

$$\overline{E}' = \frac{1}{2}\left[1 + \left(\frac{234}{236}\right)^2\right]E = 0.99E$$

Scattering on light nucleus (^{1}H) – large energy loss (good moderator) – Water used as moderator because it contains H.

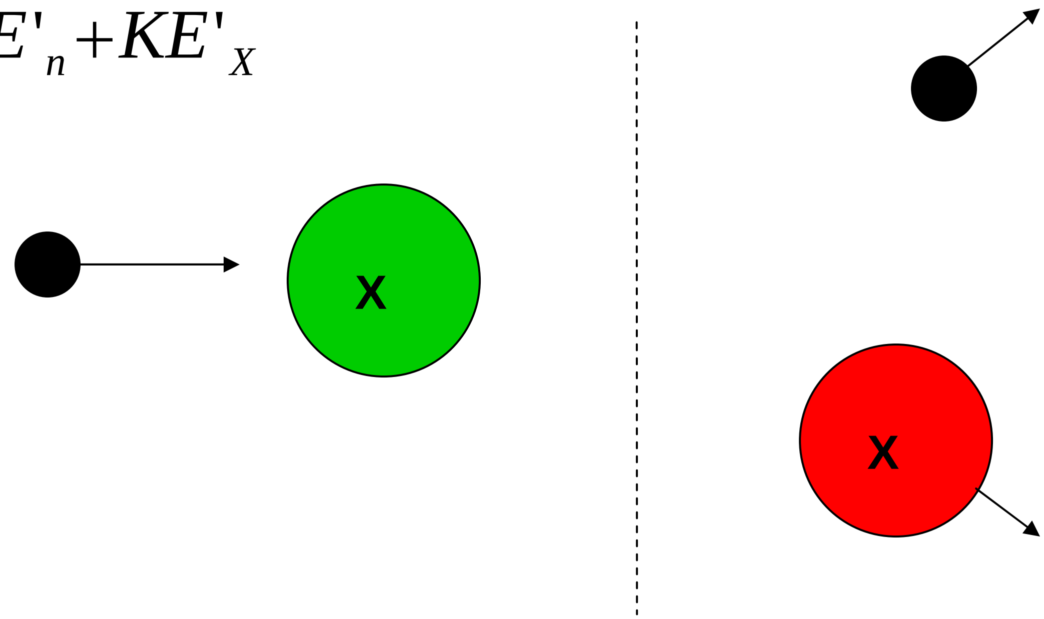
$$\overline{E}' = \frac{1}{2}\left[1 + \left(\frac{0}{2}\right)^2\right]E = 0.5E$$

Inelastic scattering

$$n + {}^{A}_{Z}X \rightarrow n' + {}^{A}_{Z}X$$

Kinetic energy is not conserved any more (total energy is)

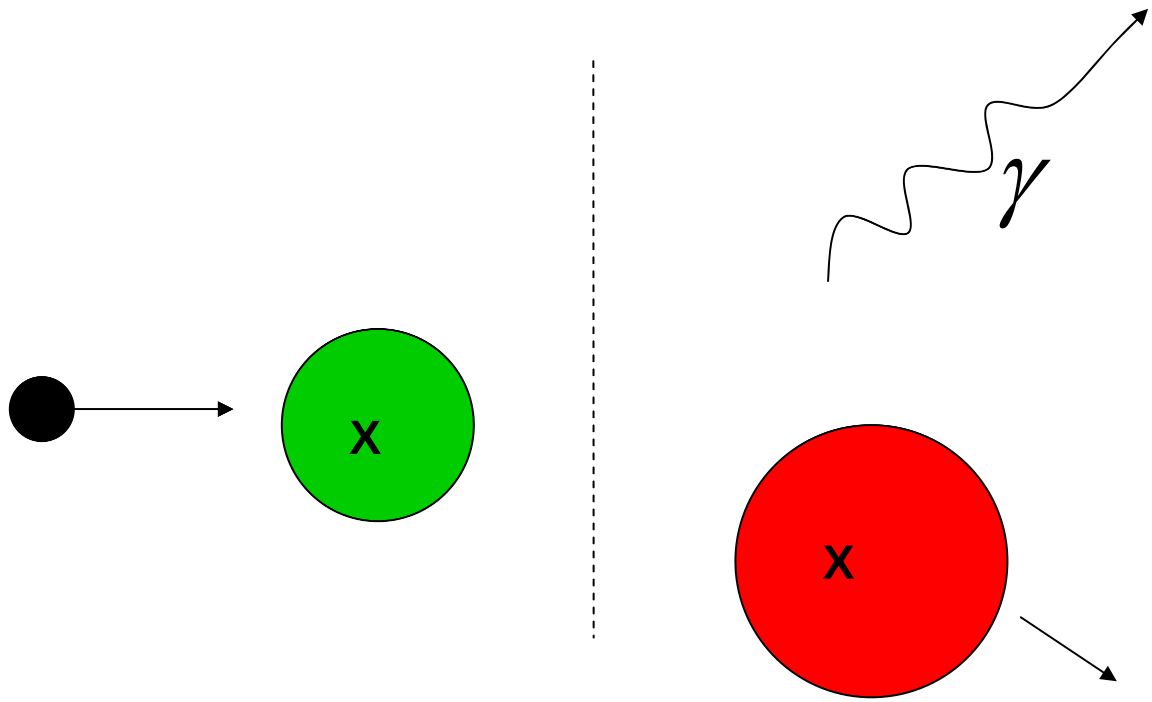
$$KE_n + KE_X > KE'_n + KE'_X$$



The incident neutron is slowed down by inelastic scattering

Radiative Capture

$$n + {}^{A}_{Z}X \rightarrow {}^{A+1}_{Z}X + \gamma$$



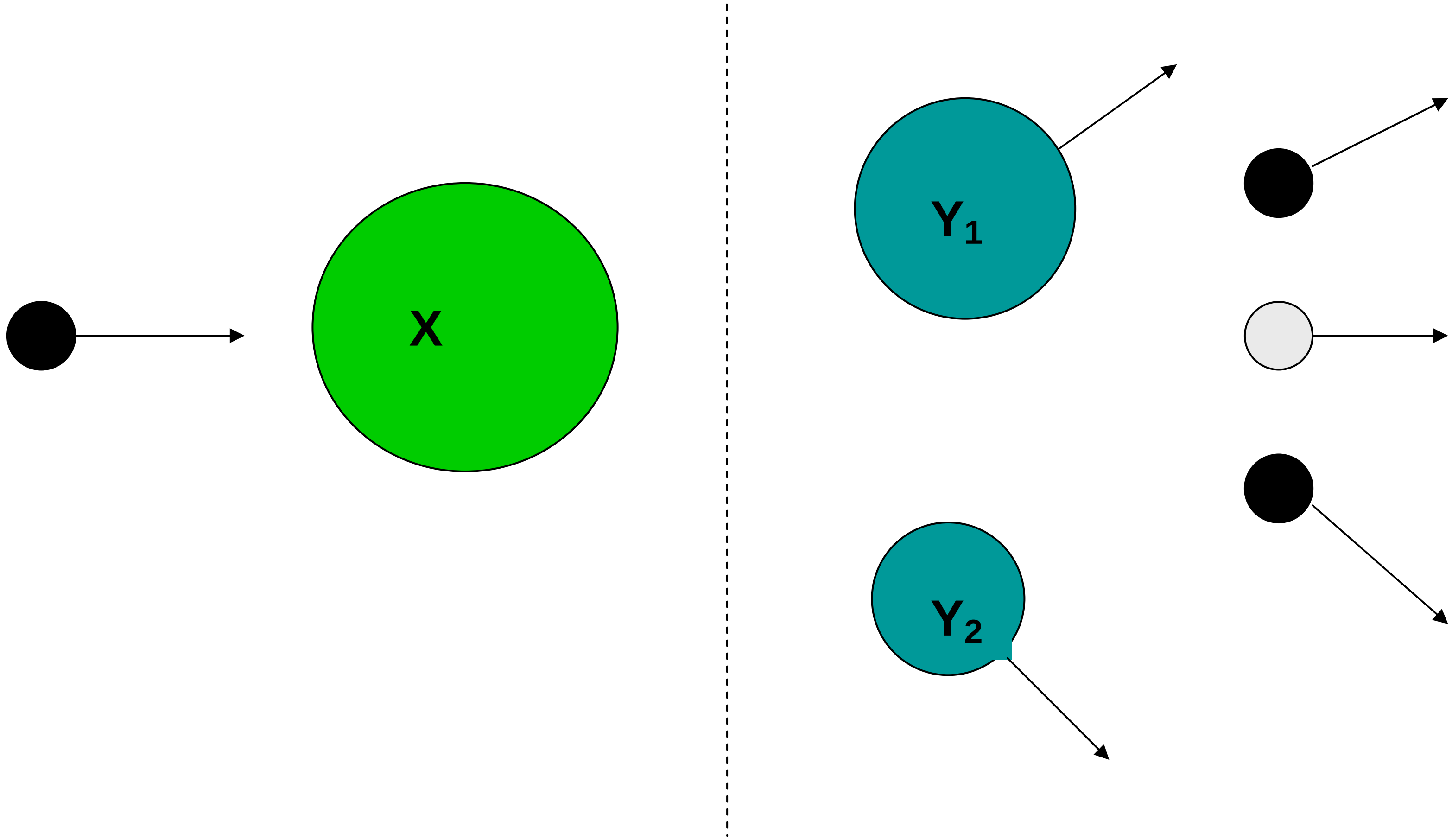
The incident neutron is absorbed (disappears) by radiative capture

Fission

$$n + {}_{Z}^{A}X \rightarrow {}_{Z_1}^{A_1}Y_1 + {}_{Z_2}^{A_2}Y_2 + \nu n + \beta + \gamma + neutrinos + approx.\ 200 MeV$$

ν = average number of neutrons ≈ 2.5

(2 to 5 neutrons can be produced)



Fission - Example

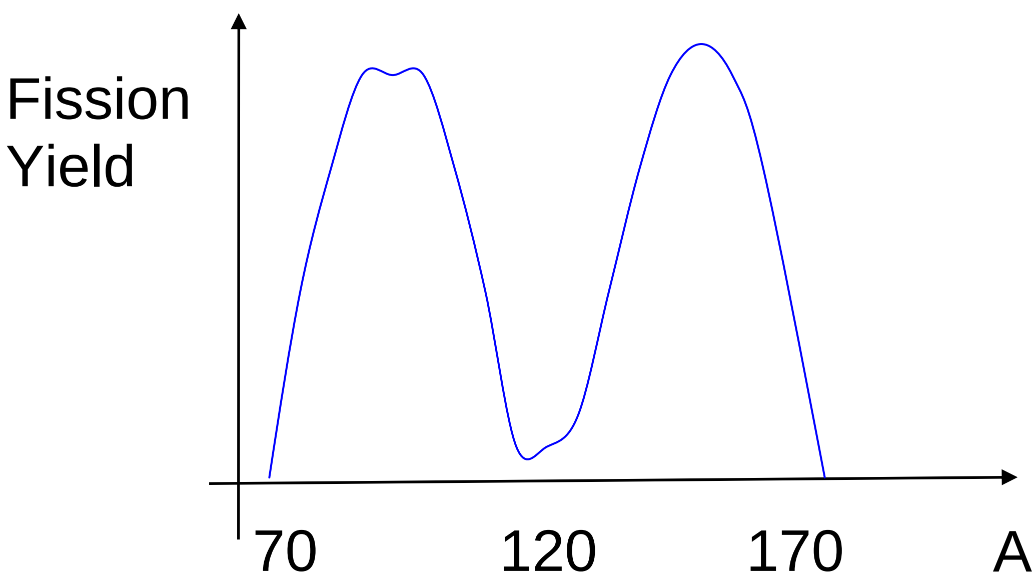
$$^{1}_{0}n + ^{235}_{92}U \rightarrow X + Y + neutrons$$

- Possible fission reactions

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2n$$

$$^{1}_{0}n + ^{235}_{92}U \rightarrow ^{132}_{50}Sn + ^{101}_{42}Mo + 3n$$

- Distribution of fragments



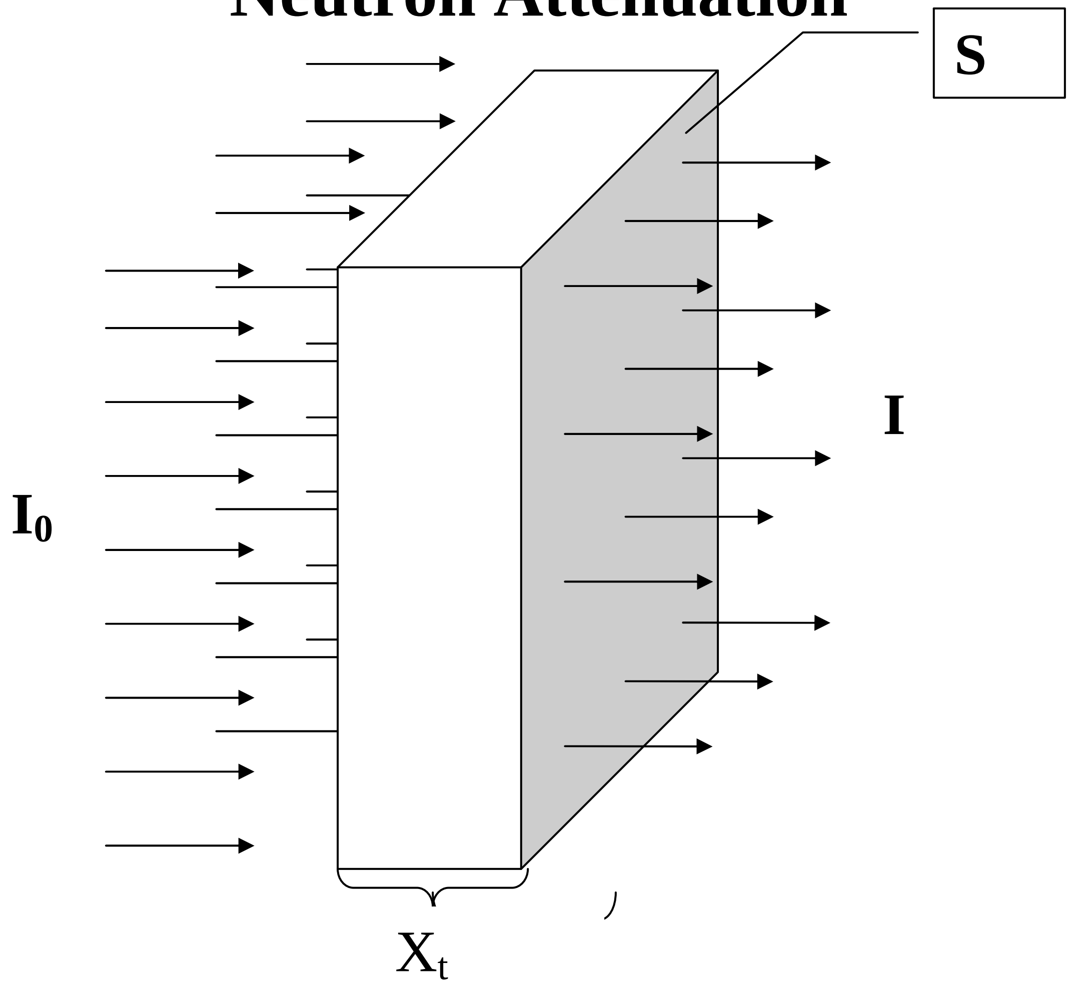
Attenuation of a Neutron Beam
(applies to any neutral particles, including photons)

Neutron Attenuation

Attenuation of a collimated (parallel) beam

- Consider a beam of neutrons of intensity I_0 that hits a target of thickness x_t, and a collimated detector that measures the intensity of the beam emerging from the target. The fact that the detector is collimated means that only the particles that have not interacted in any way are detected.
- The intensity is defined as the number of particles that pass through a surface S per unit time and per unit area.

Neutron Attenuation

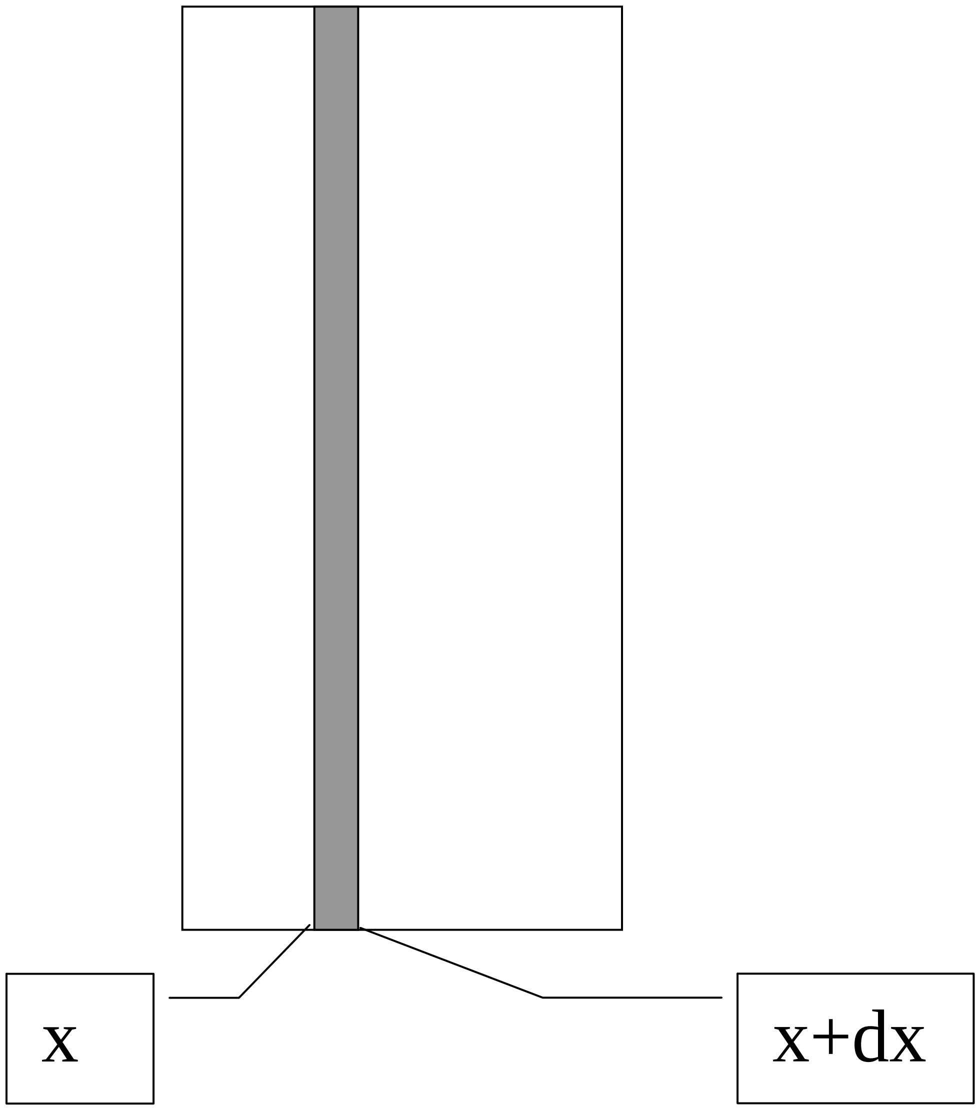


- The atom (number) density of atoms in the slab is N. (number of atoms per unit volume)
- The area of the material surface perpendicular to the beam is denoted by S.

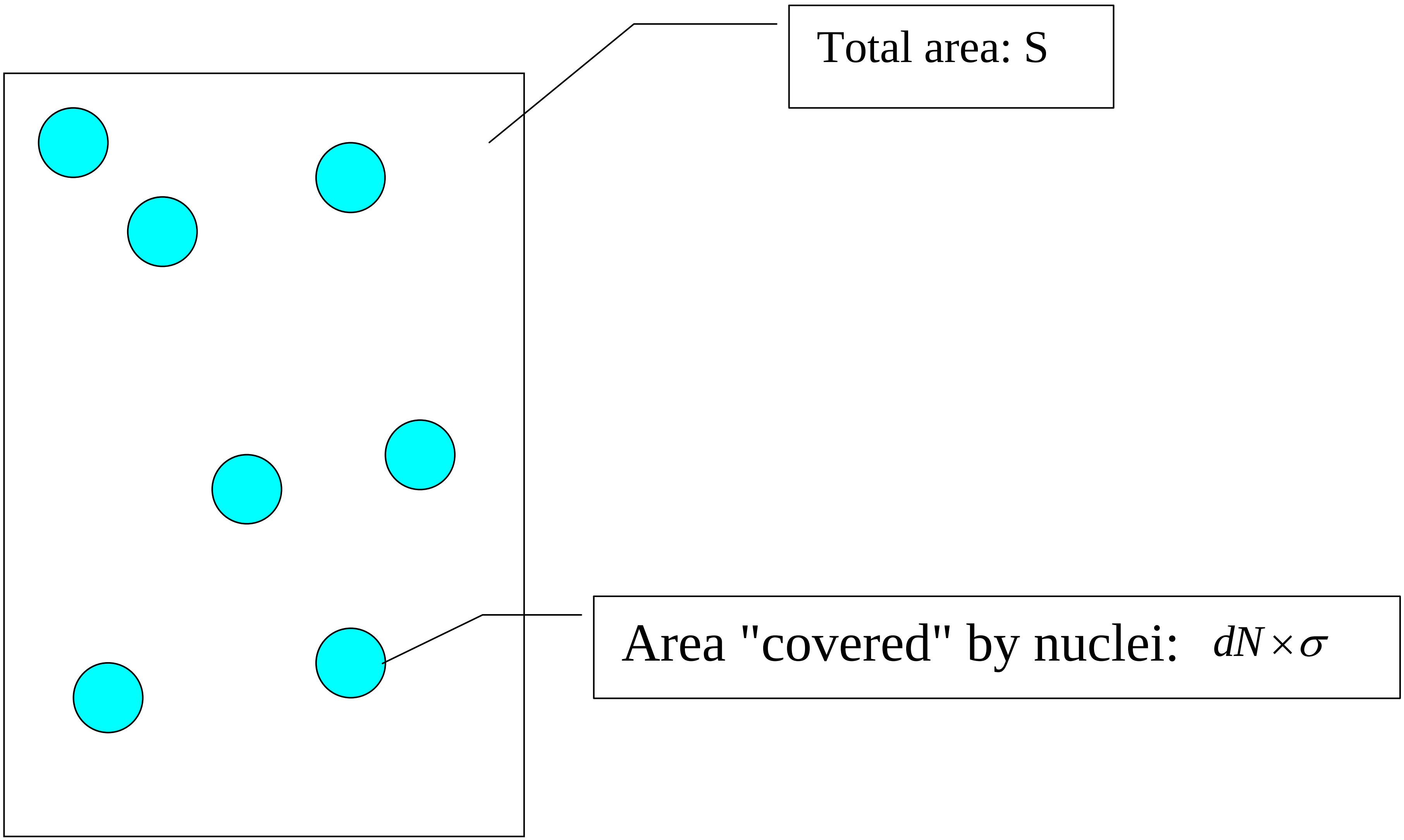
Neutron Attenuation

- Consider a thin "slice" of material, of thickness dx situated at depth x in the material.
- Consider each nucleus can be represented as a hard ball of radius, r_a , and with a corresponding cross-section area $\sigma = \pi r_a^2$
  - Also called "microscopic cross section"
- The number of nuclei (atoms) in the slice is $dN = NSdx$
  - where *N* is the atom density
- Consider neutrons to be infinitely small (points)

Thin slice of material



View of the dx slice from the neutrons' perspective



Attenuation of a collimated beam of neutrons

The probability that a neutron "hits" a nucleus equals the ratio between the area "covered" by nuclei and the total area of the slice.

Let $N_n(x)$ be the total number of neutrons that enter the slice over a time Δt

$$N_n(x) = I(x)S\Delta t$$

Let $N_n(x+dx)$ be the total number of neutrons that exit the slice dx over a time Δt (w/o undergoing any collision)

$$N_n(x+dx) = I(x+dx)S\Delta t$$

The probability of a neutron interacting with a nucleus is:

$$dP_{coll} = \frac{dN \times \sigma}{S} = \frac{N \times S \times dx \times \sigma}{S} = N \times \sigma \times dx = \Sigma \times dx$$

Macroscopic Cross Section

$$\Sigma = N \times \sigma$$

(units of cm^{-1})

Attenuation of a collimated beam of neutrons

Number of neutrons that interact and are therefore removed from the beam:

$$N_n(x) - N_n(x+dx) = -dN_n = N_n \times dP_{coll} = N_n \times \Sigma \times dx$$

$N_n(x)$ represents the number of neutrons that “survive” to cross a plane located at position x.

Setting up the differential equation

- Account for the fact that the number of neutrons that interact represent the change in the number of neutrons that exit the slice, with a negative sign

$$-dN_n(x) = N_n(x) \times \Sigma \times dx$$

- Solution

$$N_n(x) = N_{n0} e^{-\Sigma x}$$

- N_{n0} is the number of neutrons entering the material at x=0
- $N_n(x)$ is the number of neutrons that “make it” to depth x without interacting.

Attenuation of a collimated beam of neutrons

Given that

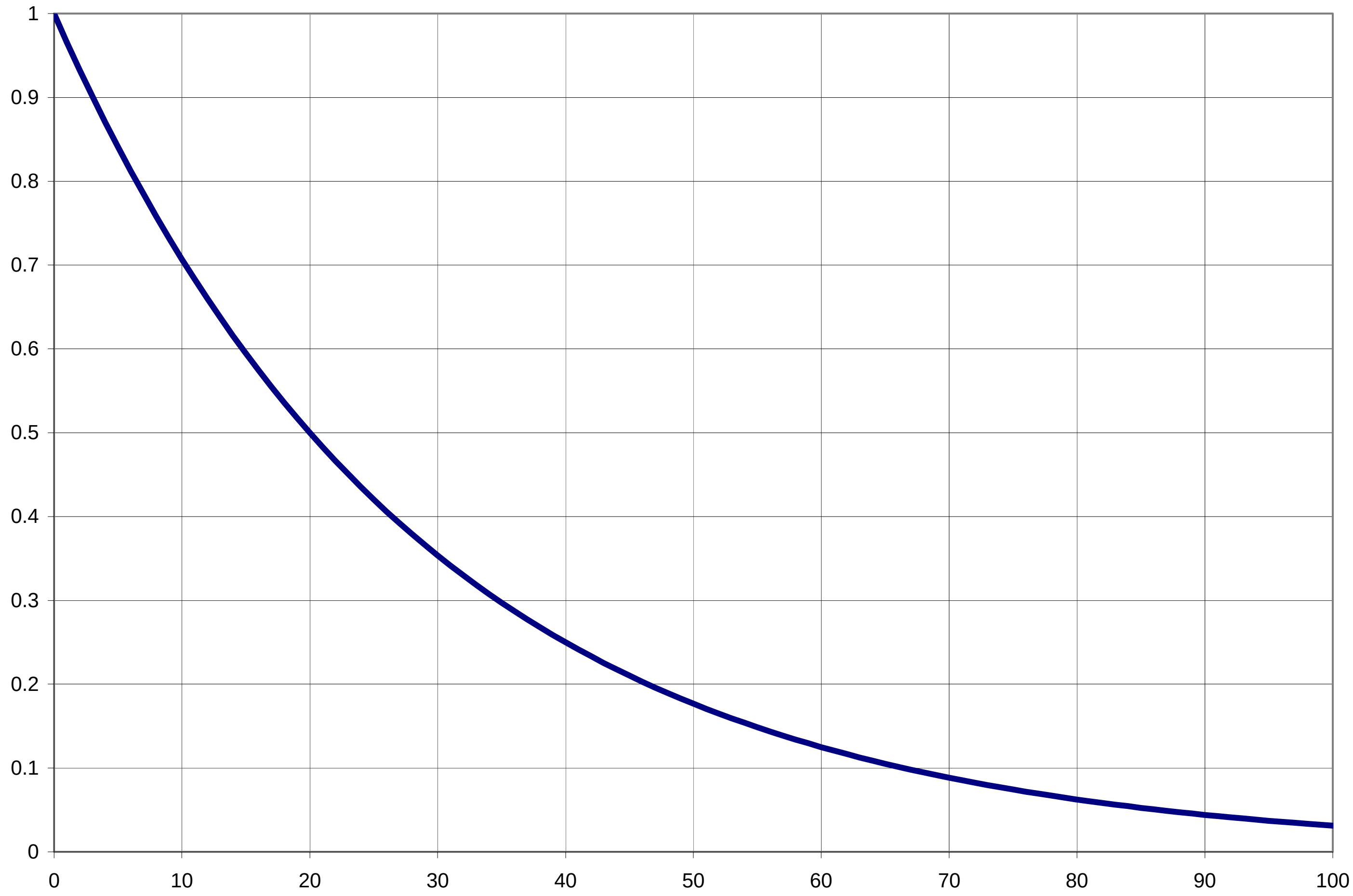
$$I = \frac{N_n}{S \times \Delta t}$$

We also have

$$I(x) = \frac{N_n(x)}{S \times \Delta t} = \frac{N_{n0} e^{-\Sigma x}}{S \times \Delta t} = I_0 e^{-\Sigma x}$$

Exponential attenuation (of a collimated beam of neutrons)

$N_n(x)/N_{n0}$
or
$I(x)/I_0$



Reaction (Collision) Rate Density

- For a thin slice of thickness dx, the volumetric reaction rate (density) is:

$$R \equiv F = \frac{\text{number of collisions}}{\text{time} \times \text{volume}} =$$

$$= \frac{-dN_n}{\Delta t \times S \times dx} = \frac{N_n(x) \times \Sigma \times dx}{\Delta t \times S \times dx} =$$

$$= \frac{N_n(x)}{\Delta t \times S}\Sigma = \frac{I(x) \times S \times \Delta t}{\Delta t \times S}\Sigma = I(x)\Sigma$$

Attenuation of a Photon Beam

Photon Attenuation

- Same reasoning as for neutrons, but with specific features
  - Instead of the density of nuclei we talk about the density of atoms, also equal to N. That is because photons interact with atoms as a whole, not just with nuclei.
- The product $N\sigma$ is called ***attenuation coefficient*** (as opposed to macroscopic cross section) and denoted by μ (as opposed to Σ).

$$\mu = N\sigma$$

(units of cm^{-1})

Attenuation:

$$I(x) = I_0 e^{-\mu x}$$

Photon Reaction Rate

$$R \equiv F = I(x)\mu$$

Neutron Beam Intensity

- Let n(x) be the neutron density (neutrons/cm^3)
- Consider monoenergetic neutrons (All have the same speed)
- Let v be the speed of neutrons.
- Consider a thin "slice" of beam of thickness dx, that crosses surface S.
- There are $dN_n = nSdx$ neutrons in this slice (box).
- It takes the neutrons in the slice time $dt = \frac{dx}{\mathrm{v}}$ to cross surface S.
- The beam intensity is therefore:

$$I = \frac{dN_n}{Sdt} = \frac{nSdx}{S\frac{dx}{\mathrm{v}}} = n\mathrm{v}$$