# Mathematics and Modelling Refresher Course Practice Problems

# Algebra

## Exponential and Logarithmic Functions

**1-1** Solve $2^x = 30$. ANS $x = \ln 30/\ln 2 = 4.91$

**1-2** Solve $5^{(x-1)} = 3^x$. ANS $x = \ln 5/\ln(5/3) = 3.15$

**1-3** Solve $7^x = 5^{x^2}$. ANS $x = 0,\ \ln 7/\ln 5$

**1-4** Solve $2\ln(\sqrt{x}) - \ln(6x-1) = 0$. ANS $x = \frac{1}{5}$

**1-5** Solve $1 - \log_{10}(7-x) = \log_{10} x$. ANS $x = 2,\ 5$

**1-6** Solve $\ln x + \ln(x-1) = \ln(3x+12)$. ANS $x = 6$

**1-7** Solve $\ln x + \ln(x-1) = \ln(3x+12)$. ANS $x = 6$

**1-8** Solve $\log_2(x^2-6x) = 3 + \log_2(1-x)$. ANS $x = -4$

**1-9** The Richter scale is commonly used to measure the intensity of an earthquake. If $E$ is the energy released, measured in joules, during an earthquake then the magnitude of the earthquake is given by,

$$M = \frac{2}{3}\log_{10}\left(\frac{E}{E_0}\right),$$

where $E_0 = 10^{4.4}$ joules.

(1) If $8\times10^{14}$ joules of energy is released during an earthquake, what was the magnitude of the earthquake?

(2) How much energy will be released in an earthquake with a magnitude of 5.9?

ANS (1) 7.0 (2) $5.6\times10^{17}$

## Vectors

**1-10** Find the dot products (1) $\mathbf{u}\cdot\mathbf{v}$, (2) $\mathbf{u}\cdot\mathbf{w}$, (3) $\mathbf{w}\cdot\mathbf{v}$

$$\mathbf{u} = \{1,\ 2,\ 3\},\quad \mathbf{v} = \{2,\ 3,\ 4\},\quad \mathbf{w} = \{-1,\ 2,\ -3\}.$$

ANS (1) 20 (2) $-8$ (3) $-6$

**1-11** Find the cross products (1) $\mathbf{u}\times\mathbf{v}$, (2) $\mathbf{v}\times\mathbf{w}$, (3) $\mathbf{w}\times\mathbf{u}$

$$\mathbf{u}=\{1,\ 2,\ 3\},\quad \mathbf{v}=\{2,\ 3,\ 4\},\quad \mathbf{w}=\{-1,\ 2,\ -3\}.$$

ANS (1) $\{-1,\ 2,\ -1\}$ (2) $\{-17,\ 2,\ 7\}$ (3) $\{12,\ 0,\ -4\}$

## Linear Algebra

**1-12** Solve the following systems of equations

$$(1)\ \begin{cases} x+2y=-5 \\ 2x+y=2 \end{cases} \qquad (2)\ \begin{cases} 5x+4y=1 \\ 3x-6y=2 \end{cases} \qquad (3)\ \begin{cases} 3x-2y=12 \\ 2x+y=1 \end{cases}$$

ANS (1) $x=3,\ y=-4$ (2) $x=\frac{1}{3},\ y=-\frac{1}{6}$ (3) $x=2,\ y=-3$

**1-13** Evaluate the following determinant

$$(1)\ \begin{vmatrix} 1 & 2 & -4 \\ -2 & 2 & 1 \\ -3 & 4 & -2 \end{vmatrix} \qquad (2)\ \begin{vmatrix} 2 & 0 & 1 \\ 1 & -4 & -1 \\ -1 & 8 & 3 \end{vmatrix} \qquad (3)\ \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} \qquad (4)\ \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

ANS (1) $-14$ (2) $-4$ (3) $3abc-a^3-b^3-c^3$ (4) $(a-b)(b-c)(c-a)$

**1-14** For

$$\mathbf{A}=\begin{bmatrix} 3 & 1 & -1 \\ 2 & 0 & 3 \end{bmatrix},\qquad \mathbf{B}=\begin{bmatrix} 1 & 6 \\ 3 & -5 \\ -2 & 4 \end{bmatrix},\qquad \mathbf{C}=\begin{bmatrix} 4 & -6 \\ 1 & 2 \end{bmatrix},$$

find each of the following: (1) **AB**, (2) **BA**, (3) **BC**.

ANS $(1)\ \begin{bmatrix} 8 & 9 \\ -4 & 24 \end{bmatrix}\quad (2)\ \begin{bmatrix} 15 & 1 & 17 \\ -1 & 3 & -18 \\ 2 & -2 & 14 \end{bmatrix}\quad (3)\ \begin{bmatrix} 10 & 6 \\ 7 & -28 \\ -4 & 20 \end{bmatrix}$

**1-15** Determine the values of $\lambda$ and $\mu$ so that the following system of homogeneous linear equations has non-zero solution:

$$\begin{cases} \lambda x_1+x_2+x_3=0 \\ x_1+\mu x_2+x_3=0 \\ x_1+2\mu x_2+x_3=0 \end{cases}$$

ANS $\lambda=1$ or $\mu=0$

**1-16** Determine the values of $\lambda$ so that the following system of homogeneous linear equations has non-zero solution:

$$\begin{cases} (1-\lambda)x_1 - 2x_2 + 4x_3 = 0 \\ 2x_1 + (3-\lambda)x_2 + x_3 = 0 \\ x_1 + x_2 + (1-\lambda)x_3 = 0 \end{cases}$$

ANS $\lambda = 0,\ 2,\ \text{or}\ 3$

**1-17** Determine the eigenvalues and corresponding eigenvectors of the following matrices

(1) $\begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 4 \\ 1 & -2 \end{bmatrix}$ (3) $\begin{bmatrix} -2 & 1 & 1 \\ 0 & 2 & 0 \\ -4 & 1 & 3 \end{bmatrix}$ (4) $\begin{bmatrix} -2 & 1 & -2 \\ 1 & -2 & 2 \\ 3 & -3 & 5 \end{bmatrix}$

ANS (1) $\lambda_1 = 2,\ \lambda_2 = 4;\quad \mathbf{v}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix},\quad \mathbf{v}_2 = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix}$

(2) $\lambda_1 = -3,\ \lambda_2 = 2;\quad \mathbf{v}_1 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix},\quad \mathbf{v}_2 = \begin{Bmatrix} 4 \\ 1 \end{Bmatrix}$

(3) $\lambda_1 = -1,\ \lambda_2 = \lambda_3 = 2;\quad \mathbf{v}_1 = \begin{Bmatrix} 1 \\ 0 \\ 1 \end{Bmatrix},\quad \mathbf{v}_2 = \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix},\quad \mathbf{v}_3 = \begin{Bmatrix} 1 \\ 0 \\ 4 \end{Bmatrix}$

(4) $\lambda_1 = \lambda_2 = -1,\ \lambda_3 = 3;\quad \mathbf{v}_1 = \begin{Bmatrix} 1 \\ 1 \\ 0 \end{Bmatrix},\quad \mathbf{v}_2 = \begin{Bmatrix} -2 \\ 0 \\ 1 \end{Bmatrix},\quad \mathbf{v}_3 = \begin{Bmatrix} -1 \\ 1 \\ 3 \end{Bmatrix}$