

1. Table of Trigonometric Identities

Trigonometric Functions

- $\sin^2 \theta + \cos^2 \theta = 1$
 $\implies 1 + \tan^2 \theta = \sec^2 \theta$
 $\implies 1 + \cot^2 \theta = \csc^2 \theta$
- $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$
- $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$
- $\sin 2\theta = 2 \sin \theta \cos \theta$
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
 $= 2 \cos^2 \theta - 1$
 $= 1 - 2 \sin^2 \theta$
- $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
- $\sin A \sin B = -\frac{1}{2} [\cos(A+B) - \cos(A-B)]$
- $\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$
- $\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$
- $\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$
- $\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$
- $\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$
- $\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$
- $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$
- Euler's Formula: $e^{i\theta} = \cos \theta + i \sin \theta$
- $z^n = a(\cos \theta + i \sin \theta)$
 $\implies z = \sqrt[n]{a} \left(\cos \frac{\theta + 2k\pi}{n} + i \sin \frac{\theta + 2k\pi}{n} \right), \quad k = 0, 1, \dots, n-1$

Hyperbolic Functions

- $$\cosh^2 x - \sinh^2 x = 1$$
- $$\implies 1 - \tanh^2 x = \operatorname{sech}^2 x$$
- $$\implies \coth^2 x - 1 = \operatorname{csch}^2 x$$
- $$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$
- $$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$
- $$\sinh 2x = 2 \sinh x \cosh x$$
- $$\cosh 2x = \cosh^2 x + \sinh^2 x$$
- $$= 2 \cosh^2 x - 1$$
- $$= 1 + 2 \sinh^2 x$$

2. Table of Derivatives

$$1. \frac{d}{dx} x^n = nx^{n-1}$$

$$2. \frac{d}{dx} e^x = e^x \implies \frac{d}{dx} a^x = a^x \ln a, \quad \because a^x = e^{x \ln a}$$

$$3. \frac{d}{dx} \ln x = \frac{1}{x} \implies \frac{d}{dx} \log_a x = \frac{1}{x \ln a}, \quad \because \log_a x = \frac{\ln x}{\ln a}$$

Trigonometric Functions

$$4. \frac{d}{dx} \sin x = \cos x$$

$$5. \frac{d}{dx} \cos x = -\sin x$$

$$6. \frac{d}{dx} \tan x = \frac{1}{\cos^2 x} = \sec^2 x$$

$$7. \frac{d}{dx} \cot x = -\frac{1}{\sin^2 x} = -\operatorname{csc}^2 x$$

$$8. \frac{d}{dx} \sec x = \frac{\sin x}{\cos^2 x} = \tan x \sec x$$

$$9. \frac{d}{dx} \csc x = -\frac{\cos x}{\sin^2 x} \\ = -\cot x \csc x$$

Hyperbolic Functions

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \tanh x = \frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$\frac{d}{dx} \coth x = -\frac{1}{\sinh^2 x} = -\operatorname{csch}^2 x$$

$$\frac{d}{dx} \operatorname{sech} x = -\frac{\sinh x}{\cosh^2 x} \\ = -\tanh x \operatorname{sech} x$$

$$\frac{d}{dx} \operatorname{csch} x = -\frac{\cosh x}{\sinh^2 x} \\ = -\coth x \operatorname{csch} x$$

Inverse Trigonometric Functions

$$10. \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$11. \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$12. \frac{d}{dx} \sec^{-1} x = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx} \cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cot^{-1} x = -\frac{1}{1+x^2}$$

$$\frac{d}{dx} \csc^{-1} x = -\frac{1}{x\sqrt{x^2-1}}$$

3. Table of Integrals

$$1. \int x^n dx = \frac{x^{n+1}}{n+1}, \quad n \neq -1$$

$$2. \int \frac{1}{x} dx = \ln|x|, \quad x \neq 0 \qquad \int \ln x dx = x(\ln x - 1), \quad x > 0$$

$$3. \int e^{ax} dx = \frac{1}{a} e^{ax} \qquad \int b^{ax} dx = \frac{b^{ax}}{a \ln b}, \quad b > 0$$

Trigonometric Functions

$$4. \int \sin x dx = -\cos x$$

$$5. \int \cos x dx = \sin x$$

$$6. \int \tan x dx = -\ln|\cos x|$$

$$7. \int \cot x dx = \ln|\sin x|$$

$$8. \int \sec x dx = \ln|\sec x + \tan x| \\ = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| = \ln\left|\cot\left(\frac{\pi}{4} - \frac{x}{2}\right)\right|$$

$$9. \int \csc x dx = \ln|\csc x - \cot x| \\ = \ln\left|\tan\frac{x}{2}\right|$$

$$10. \int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x$$

$$11. \int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x$$

$$12. \int \tan^2 x dx = \tan x - x$$

$$13. \int \cot^2 x dx = -\cot x - x$$

$$14. \int \sec^2 x dx = \tan x$$

$$15. \int \csc^2 x dx = -\cot x$$

Hyperbolic Functions

$$\int \sinh x dx = \cosh x$$

$$\int \cosh x dx = \sinh x$$

$$\int \tanh x dx = \ln \cosh x$$

$$\int \coth x dx = \ln|\sinh x|$$

$$\int \operatorname{sech} x dx = \tan^{-1}(\sinh x)$$

$$\int \operatorname{csch} x dx = \ln\left|\tanh\frac{x}{2}\right|$$

$$\int \sinh^2 x dx = \frac{1}{4} \sinh 2x - \frac{x}{2}$$

$$\int \cosh^2 x dx = \frac{1}{4} \sinh 2x + \frac{x}{2}$$

$$\int \tanh^2 x dx = -\tanh x + \frac{1}{2} \ln\left|\frac{\tanh x + 1}{\tanh x - 1}\right|$$

$$\int \coth^2 x dx = -\coth x + \frac{1}{2} \ln\left|\frac{\coth x + 1}{\coth x - 1}\right|$$

$$\int \operatorname{sech}^2 x dx = \tanh x$$

$$\int \operatorname{csch}^2 x dx = -\coth x$$

16. $\int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax$
17. $\int x^2 \sin ax \, dx = \frac{2x}{a^2} \sin ax - \frac{a^2 x^2 - 2}{a^3} \cos ax$
18. $\int x^3 \sin ax \, dx = \frac{3a^2 x^2 - 6}{a^4} \sin ax - \frac{a^2 x^3 - 6x}{a^3} \cos ax$
19. $\int x^4 \sin ax \, dx = \frac{4a^2 x^3 - 24x}{a^4} \sin ax - \frac{a^4 x^4 - 12a^2 x^2 + 24}{a^5} \cos ax$
20. $\int x \cos ax \, dx = \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax$
21. $\int x^2 \cos ax \, dx = \frac{a^2 x^2 - 2}{a^3} \sin ax + \frac{2x}{a^2} \cos ax$
22. $\int x^3 \cos ax \, dx = \frac{a^2 x^3 - 6x}{a^3} \sin ax + \frac{3a^2 x^2 - 6}{a^4} \cos ax$
23. $\int x^4 \cos ax \, dx = \frac{a^4 x^4 - 12a^2 x^2 + 24}{a^5} \sin ax + \frac{4a^2 x^3 - 24x}{a^4} \cos ax$
24. $\int e^{bx} \sin ax \, dx = e^{bx} \frac{b \sin ax - a \cos ax}{a^2 + b^2}$
25. $\int e^{bx} \cos ax \, dx = e^{bx} \frac{a \sin ax + b \cos ax}{a^2 + b^2}$
26. $\int x e^{ax} \, dx = \frac{ax - 1}{a^2} e^{ax}$
27. $\int x^2 e^{ax} \, dx = \frac{a^2 x^2 - 2ax + 2}{a^3} e^{ax}$
28. $\int x^3 e^{ax} \, dx = \frac{a^3 x^3 - 3a^2 x^2 + 6ax - 6}{a^4} e^{ax}$
29. $\int x^4 e^{ax} \, dx = \frac{a^4 x^4 - 4a^3 x^3 + 12a^2 x^2 - 24ax + 24}{a^5} e^{ax}$
30. $\int x^5 e^{ax} \, dx = \frac{a^5 x^5 - 5a^4 x^4 + 20a^3 x^3 - 60a^2 x^2 + 120ax - 120}{a^6} e^{ax}$
31. $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a > 0$
32. $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln \frac{a+x}{a-x} \quad \text{or} \quad \frac{1}{a} \tanh^{-1} \frac{x}{a}, \quad |x| < |a|$

$$33. \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \ln \frac{x-a}{x+a} \quad \text{or} \quad -\frac{1}{a} \coth^{-1} \frac{x}{a}, \quad |x| > |a|$$

$$34. \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} \quad \text{or} \quad -\cos^{-1} \frac{x}{a}$$

$$35. \int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1} \frac{x}{a} \quad \text{or} \quad \ln|x + \sqrt{x^2 - a^2}|$$

$$36. \int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1} \frac{x}{a} \quad \text{or} \quad \ln(x + \sqrt{x^2 + a^2})$$

$$37. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$$

$$38. \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \sinh^{-1} \frac{x}{a}$$

$$\text{or} \quad \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \ln(x + \sqrt{x^2 + a^2})$$

$$39. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \frac{x}{a}$$

$$\text{or} \quad \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln|x + \sqrt{x^2 - a^2}|$$

$$40. \int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \cosh^{-1} \frac{a}{x} \quad \text{or} \quad -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$41. \int \frac{1}{x\sqrt{x^2 + a^2}} dx = -\frac{1}{a} \sinh^{-1} \frac{a}{x} \quad \text{or} \quad -\frac{1}{a} \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|$$

$$42. \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \cos^{-1} \frac{a}{x} \quad \text{or} \quad \frac{1}{a} \sec^{-1} \frac{x}{a}$$

$$43. \int \frac{1}{\sqrt{2ax - x^2}} dx = \cos^{-1} \left(1 - \frac{x}{a} \right) \quad \text{or} \quad \sin^{-1} \left(\frac{x}{a} - 1 \right)$$

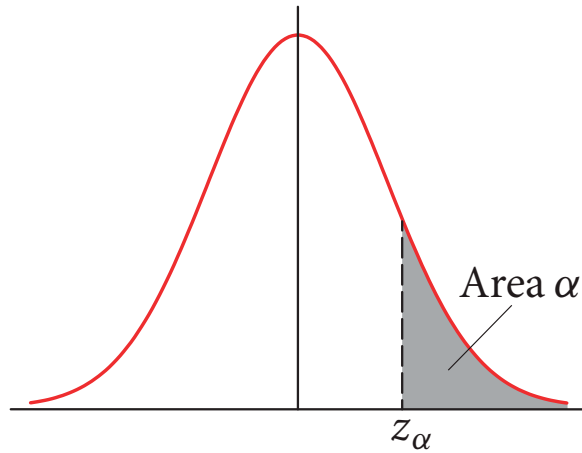
$$44. \int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x}{a} - 1 \right)$$

$$45. \int_0^{\frac{\pi}{2}} \begin{cases} \sin^n \theta \\ \cos^n \theta \end{cases} d\theta = \frac{(n-1)!!}{n!!} \times \begin{cases} \frac{1}{2}\pi, & \text{if } n \text{ is an even integer} \\ 1, & \text{if } n \text{ is an odd integer} \end{cases}$$

$$46. \int_0^{\frac{\pi}{2}} \sin^n \theta \cos^m \theta d\theta = \frac{(n-1)!! (m-1)!!}{(n+m)!!} \times \begin{cases} \frac{1}{2}\pi, & n, m \text{ even integers} \\ 1, & \text{otherwise} \end{cases}$$

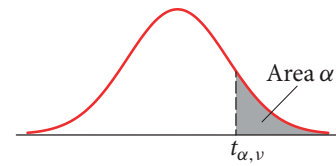
$$\nabla n!! = \begin{cases} n \cdot (n-2) \cdots 5 \cdot 3 \cdot 1, & n > 0 \text{ odd integer} \\ n \cdot (n-2) \cdots 6 \cdot 4 \cdot 2, & n > 0 \text{ even integer} \\ 1, & n = 0 \end{cases}$$

5. Table of Critical Normal Deviate Values



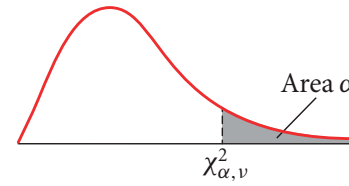
Upper-Tail Area α	Normal Deviate z_α
0.10	1.28
0.05	1.64
0.025	1.96
0.01	2.33
0.005	2.57
0.001	3.08
0.0005	3.30

6. Table of Student t Distribution: $t_{\alpha, \nu}$



ν	Upper-Tail Area α									
	.4	.25	.1	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

7. Table of Chi-Square Distribution $\chi^2_{\alpha, \nu}$



Upper-Tail Area α

ν	0.99	0.98	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.02	0.01	0.001	ν
1	0.000157	0.000628	0.00393	0.0158	0.0642	0.148	0.455	1.074	1.642	2.706	3.841	5.412	6.635	10.827	1
2	0.0201	0.0404	0.103	0.211	0.446	0.713	1.386	2.408	3.219	4.605	5.991	7.824	9.210	13.815	2
3	0.115	0.185	0.352	0.584	1.005	1.424	2.366	3.665	4.642	6.251	7.815	9.837	11.345	16.268	3
4	0.297	0.429	0.711	1.064	1.649	2.195	3.357	4.878	5.989	7.779	9.488	11.668	13.277	18.465	4
5	0.554	0.752	1.145	1.610	2.343	3.000	4.351	6.064	7.289	9.236	11.070	13.388	15.086	20.517	5
6	0.872	1.134	1.635	2.204	3.070	3.828	5.348	7.231	8.558	10.645	12.592	15.033	16.812	22.457	6
7	1.239	1.564	2.167	2.833	3.822	4.671	6.346	8.383	9.803	12.017	14.067	16.622	18.475	24.322	7
8	1.646	2.032	2.733	3.490	4.594	5.527	7.344	9.524	11.030	13.362	15.507	18.168	20.090	26.125	8
9	2.088	2.532	3.325	4.168	5.380	6.393	8.343	10.656	12.242	14.684	16.919	19.679	21.666	27.877	9
10	2.558	3.059	3.940	4.865	6.179	7.267	9.342	11.781	13.442	15.987	18.307	21.161	23.209	29.588	10
11	3.053	3.609	4.575	5.578	6.989	8.148	10.341	12.899	14.631	17.275	19.675	22.618	24.725	31.264	11
12	3.571	4.178	5.226	6.304	7.807	9.034	11.340	14.011	15.812	18.549	21.026	24.054	26.217	32.909	12
13	4.107	4.765	5.892	7.042	8.634	9.926	12.340	15.119	16.985	19.812	22.362	25.472	27.688	34.528	13
14	4.660	5.368	6.571	7.790	9.467	10.821	13.339	16.222	18.151	21.064	23.685	26.873	29.141	36.123	14
15	5.229	5.985	7.261	8.547	10.307	11.721	14.339	17.322	19.311	22.307	24.996	28.259	30.578	37.697	15
16	5.812	6.614	7.962	9.312	11.152	12.624	15.338	18.418	20.465	23.542	26.296	29.633	32.000	39.252	16
17	6.408	7.255	8.672	10.085	12.002	13.531	16.338	19.511	21.615	24.769	27.587	30.995	33.409	40.790	17
18	7.015	7.906	9.390	10.865	12.857	14.440	17.338	20.601	22.760	25.989	28.869	32.346	34.805	42.312	18
19	7.633	8.567	10.117	11.651	13.716	15.352	18.338	21.689	23.900	27.204	30.144	33.687	36.191	43.820	19
20	8.260	9.237	10.851	12.443	14.578	16.266	19.337	22.775	25.038	28.412	31.410	35.020	37.566	45.315	20
21	8.897	9.915	11.591	13.240	15.445	17.182	20.337	23.858	26.171	29.615	32.671	36.343	38.932	46.797	21
22	9.542	10.600	12.338	14.041	16.314	18.101	21.337	24.939	27.301	30.813	33.924	37.659	40.289	48.268	22
23	10.196	11.293	13.091	14.848	17.187	19.021	22.337	26.018	28.429	32.007	35.172	38.968	41.638	49.728	23
24	10.856	11.992	13.848	15.659	18.062	19.943	23.337	27.096	29.553	33.196	36.415	40.270	42.980	51.179	24
25	11.524	12.697	14.611	16.473	18.940	20.867	24.337	28.172	30.675	34.382	37.652	41.566	44.314	52.620	25
26	12.198	13.409	15.379	17.292	19.820	21.792	25.336	29.246	31.795	35.563	38.885	42.856	45.642	54.052	26
27	12.879	14.125	16.151	18.114	20.703	22.719	26.336	30.319	32.912	36.741	40.113	44.140	46.963	55.476	27
28	13.565	14.847	16.928	18.939	21.588	23.647	27.336	31.391	34.027	37.916	41.337	45.419	48.278	56.893	28
29	14.256	15.574	17.708	19.768	22.475	24.577	28.336	32.461	35.139	39.087	42.557	46.693	49.588	58.302	29
30	14.953	16.306	18.493	20.599	23.364	25.508	29.336	33.530	36.250	40.256	43.773	47.962	50.892	59.703	30

8. Table of Laplace Transforms

Properties of Laplace Transform $\mathcal{L}\{f(t)\} = F(s)$

1. Laplace Transform of Derivatives

$$\mathcal{L}\{f'(t)\} = sF(s) - f(0)$$

$$\mathcal{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$$

... ..

$$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \dots - sf^{(n-2)}(0) - f^{(n-1)}(0)$$

2. Laplace Transform of Integrals

$$\mathcal{L}\left\{\int_0^t \dots \int_0^t f(u) (du)^n\right\} = \frac{F(s)}{s^n}$$

3. Property of Shifting

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

4. Property of Differentiation

$$\mathcal{L}\{t^n f(t)\} = (-1)^n \frac{d^n F(s)}{ds^n}, \quad n = 1, 2, \dots$$

5. Property of Integration

$$\mathcal{L}\left\{\frac{f(t)}{t^n}\right\} = \int_s^\infty \dots \int_s^\infty F(s) (ds)^n, \quad n = 1, 2, \dots$$

6. Convolution Integral

$$\mathcal{L}\left\{\int_0^t f(u)g(t-u)du\right\} = F(s)G(s)$$

7. Heaviside Function

$$\mathcal{L}\{H(t-a)\} = \frac{1}{s} e^{-as}, \quad \mathcal{L}\{f(t-a)H(t-a)\} = e^{-as}F(s)$$

8. Dirac Delta Function

$$\mathcal{L}\{\delta(t-a)\} = e^{-as}, \quad \mathcal{L}\{f(t)\delta(t-a)\} = e^{-as}f(a)$$

	$f(t)$	$\mathcal{L}\{f(t)\} = F(s)$
1.	1	$\frac{1}{s}, \quad s > 0$
2.	$t^n, \quad n = 1, 2, \dots$	$\frac{n!}{s^{n+1}}, \quad s > 0$
3.	$t^\nu, \quad \nu > -1$	$\frac{\Gamma(\nu+1)}{s^{\nu+1}}, \quad s > 0$
4.	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}, \quad s > 0$
5.	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}, \quad s > 0$
6.	$t \sin \omega t$	$\frac{2\omega s}{(s^2 + \omega^2)^2}, \quad s > 0$
7.	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}, \quad s > 0$
8.	e^{at}	$\frac{1}{s - a}, \quad s > a$
9.	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}, \quad s > \omega $
10.	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}, \quad s > \omega $
11.	$\sinh^2 \omega t$	$\frac{2\omega^2}{s(s^2 - 4\omega^2)}, \quad s > 2 \omega $
12.	$\cosh^2 \omega t$	$\frac{s^2 - 2\omega^2}{s(s^2 - 4\omega^2)}, \quad s > 2 \omega $
13.	$t^{\nu-1} \sinh \omega t, \quad \operatorname{Re}(\nu) > -1, \nu \neq 0$	$\frac{\Gamma(\nu)}{2} \left[\frac{1}{(s-\omega)^\nu} - \frac{1}{(s+\omega)^\nu} \right], \quad s > \omega $
14.	$t^{\nu-1} \cosh \omega t, \quad \operatorname{Re}(\nu) > 0$	$\frac{\Gamma(\nu)}{2} \left[\frac{1}{(s-\omega)^\nu} + \frac{1}{(s+\omega)^\nu} \right], \quad s > \omega $
15.	$f(at), \quad a > 0$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
16.	$\int_t^\infty \frac{f(u)}{u} du$	$\frac{1}{s} \int_0^s F(u) du$

9. Table of Inverse Laplace Transforms

Properties of Inverse Laplace Transform $\mathcal{L}^{-1}\{F(s)\} = f(t)$

1. Property of Shifting

$$\mathcal{L}^{-1}\{F(s-a)\} = e^{at}f(t)$$

2. Property of Differentiation

$$\mathcal{L}^{-1}\left\{\frac{d^n F(s)}{ds^n}\right\} = (-1)^n t^n f(t), \quad n = 1, 2, \dots$$

3. Property of Integration

$$\mathcal{L}^{-1}\left\{\int_s^\infty \dots \int_s^\infty F(s) (ds)^n\right\} = \frac{f(t)}{t^n}, \quad n = 1, 2, \dots$$

4. Convolution Integral

$$\mathcal{L}^{-1}\{F(s)G(s)\} = \int_0^t f(u)g(t-u)du = \int_0^t g(u)f(t-u)du$$

5. Heaviside Function

$$\mathcal{L}^{-1}\{e^{-as}F(s)\} = f(t-a)H(t-a)$$

6. Dirac Delta Function

$$\mathcal{L}^{-1}\{1\} = \delta(t), \quad \mathcal{L}^{-1}\{e^{-as}\} = \delta(t-a)$$

	$F(s)$	$\mathcal{L}^{-1}\{F(s)\} = f(t)$
1.	$\frac{\omega}{s^2 + \omega^2}$	$\sin \omega t$
2.	$\frac{s}{s^2 + \omega^2}$	$\cos \omega t$
3.	$\frac{1}{s^n}, \quad n = 1, 2, \dots$	$\frac{1}{(n-1)!} t^{n-1}$
4.	$\frac{1}{s^{n+\frac{1}{2}}}, \quad n = 0, 1, \dots$	$\frac{2^{2n} n!}{\sqrt{\pi} (2n)!} t^{n-\frac{1}{2}}$
5.	$\frac{1}{(s-a)(s-b)}, \quad a \neq b$ $\frac{1}{s^2 - a^2}$	$\frac{1}{a-b} (e^{at} - e^{bt})$ $\frac{1}{a} \sinh at$
6.	$\frac{s}{(s-a)(s-b)}, \quad a \neq b$ $\frac{s}{s^2 - a^2}$	$\frac{1}{a-b} (ae^{at} - be^{bt})$ $\cosh at$
7.	$\frac{1}{(s-a)(s-b)(s-c)}, \quad a \neq b \neq c$	$-\frac{(b-c)e^{at} + (c-a)e^{bt} + (a-b)e^{ct}}{(a-b)(b-c)(c-a)}$
8.	$\frac{1}{s^4 + 4a^4}$	$\frac{1}{4a^3} (\sin at \cosh at - \cos at \sinh at)$
9.	$\frac{s}{s^4 + 4a^4}$	$\frac{1}{2a^2} \sin at \sinh at$
10.	$\frac{s^2}{s^4 + 4a^4}$	$\frac{1}{2a} (\sin at \cosh at + \cos at \sinh at)$
11.	$\frac{s^3}{s^4 + 4a^4}$	$\cos at \cosh at$
12.	$\frac{1}{s(s^4 + 4a^4)}$	$\frac{1 - \cos at \cosh at}{4a^4}$
13.	$\frac{1}{s^2(s^4 + 4a^4)}$	$\frac{2at - (\sin at \cosh at + \cos at \sinh at)}{8a^5}$
14.	$\frac{1}{s^3(s^4 + 4a^4)}$	$\frac{a^2 t^2 - \sin at \sinh at}{8a^6}$
15.	$\frac{1}{s^4(s^4 + 4a^4)}$	$\frac{2a^3 t^3 - 3(\sin at \cosh at - \cos at \sinh at)}{48a^7}$

	$F(s)$	$\mathcal{L}^{-1}\{F(s)\} = f(t)$
16.	$\frac{1}{(s^2 + a^2)^2}$	$\frac{1}{2a^3} (\sin at - at \cos at)$
17.	$\frac{s}{(s^2 + a^2)^2}$	$\frac{1}{2a} t \sin at$
18.	$\frac{1}{(s^2 + a^2)^3}$	$\frac{1}{8a^5} [(3 - a^2 t^2) \sin at - 3at \cos at]$
19.	$\frac{s}{(s^2 + a^2)^3}$	$\frac{1}{8a^3} t (\sin at - at \cos at)$
20.	$\frac{1}{(s^2 + a^2)(s^2 + b^2)}$	$\frac{a \sin bt - b \sin at}{ab(a^2 - b^2)}$
21.	$\frac{s}{(s^2 + a^2)(s^2 + b^2)}$	$\frac{\cos bt - \cos at}{a^2 - b^2}$
22.	$\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$	$\frac{a \sin at - b \sin bt}{a^2 - b^2}$
23.	$\frac{s^3}{(s^2 + a^2)(s^2 + b^2)}$	$\frac{a^2 \cos at - b^2 \cos bt}{a^2 - b^2}$
24.	$\frac{1}{s^4 - a^4}$	$\frac{1}{2a^3} (\sinh at - \sin at)$
25.	$\frac{s}{s^4 - a^4}$	$\frac{1}{2a^2} (\cosh at - \cos at)$
26.	$\frac{s^2}{s^4 - a^4}$	$\frac{1}{2a} (\sinh at + \sin at)$
27.	$\frac{s^3}{s^4 - a^4}$	$\frac{1}{2} (\cosh at + \cos at)$
28.	$\frac{1}{s^3 + 8a^3}$	$\frac{e^{at}(\sqrt{3} \sin \sqrt{3}at - \cos \sqrt{3}at) + e^{-2at}}{12a^2}$
29.	$\frac{s}{s^3 + 8a^3}$	$\frac{e^{at}(\sqrt{3} \sin \sqrt{3}at + \cos \sqrt{3}at) - e^{-2at}}{6a}$
30.	$\frac{s^2}{s^3 + 8a^3}$	$\frac{2e^{at} \cos \sqrt{3}at + e^{-2at}}{3}$